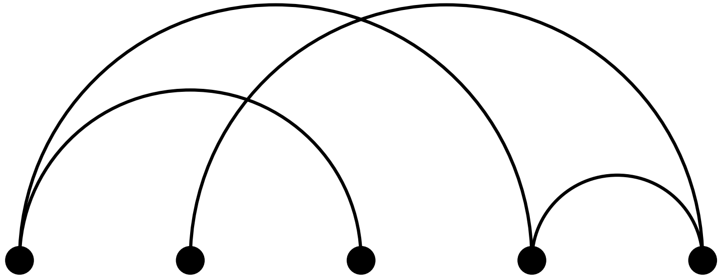
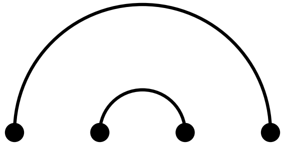
Karlsruhe Institute of Technology

Linear Layouts of Complete Graphs

GD 2021 · September 16

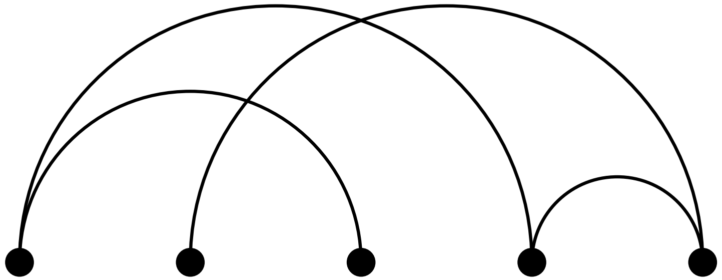
Stefan Felsner, **Laura Merker**, Torsten Ueckerdt, and Pavel Valtr

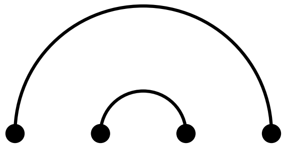


Queue:  forbidden

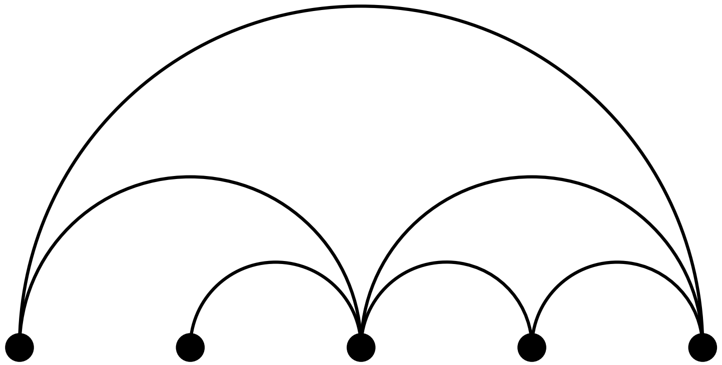
qn(G) = min k s.t. there is a vertex ordering and a partition of the edges into k queues

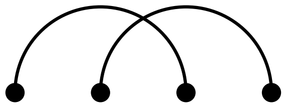
Institute of Theoretical Informatics
Algorithmics Group



Queue:  forbidden

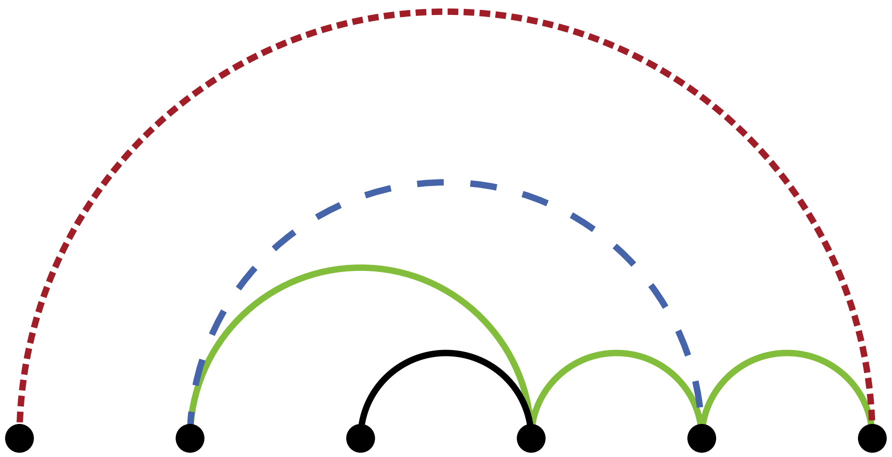
qn(G) = min k s.t. there is a vertex ordering and a partition of the edges into k queues



Page:  forbidden

pn(G) = min k s.t. there is a vertex ordering and a partition of the edges into k pages

Institute of Theoretical Informatics
Algorithmics Group

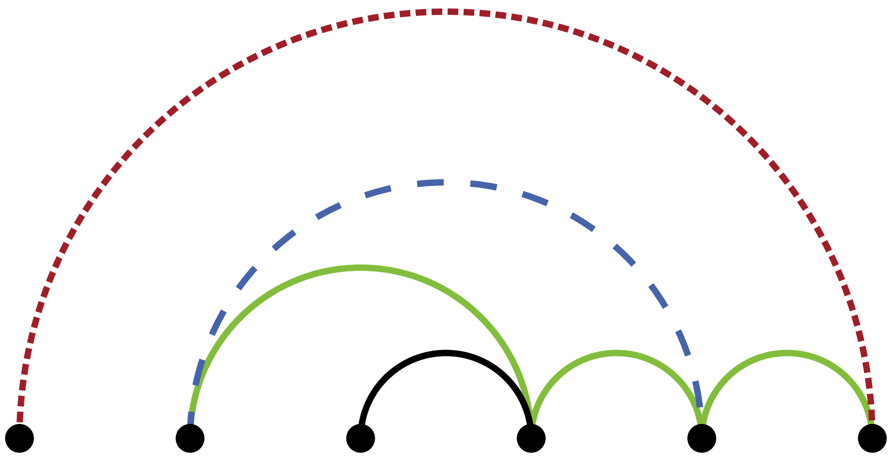


4 Queues

(Global) Queue Number

qn(G) = min k s.t. there is a partition of the edges into k queues

Institute of Theoretical Informatics
Algorithmics Group



2 Queues at each vertex

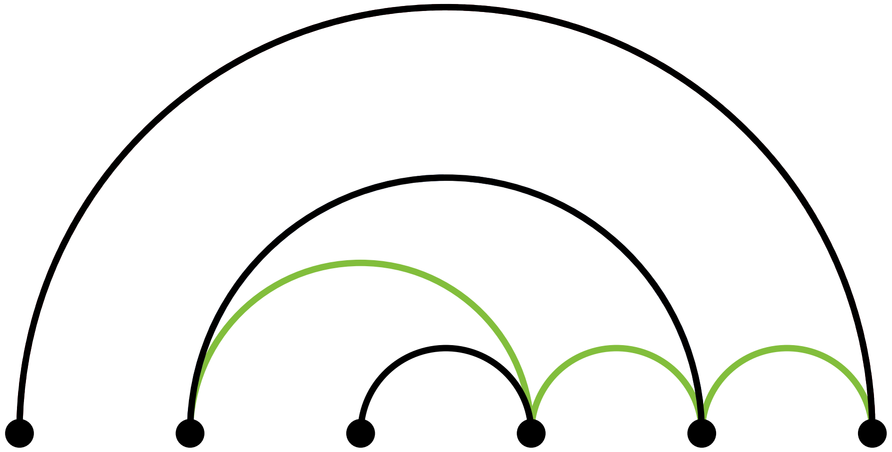
(Global) Queue Number

$\mathrm{qn}(G)$ = min k s.t. there is a partition of the edges into k queues

Local Queue Number

$\mathrm{qn}_\ell(G)$ = min k s.t. each vertex has incident edges in at most k queues

Institute of Theoretical Informatics
Algorithmics Group



Union Queue: Vertex-disjoint union of queues

(Global) Queue Number

$\mathrm{qn}(G)$ = min k s.t. there is a partition of the edges into k queues

Union Queue Number

$\mathrm{qn}_u(G)$ = min k s.t. there is a partition of the edges into k union queues

Local Queue Number

$\mathrm{qn}_\ell(G)$ = min k s.t. each vertex has incident edges in at most k queues

Complete Graphs

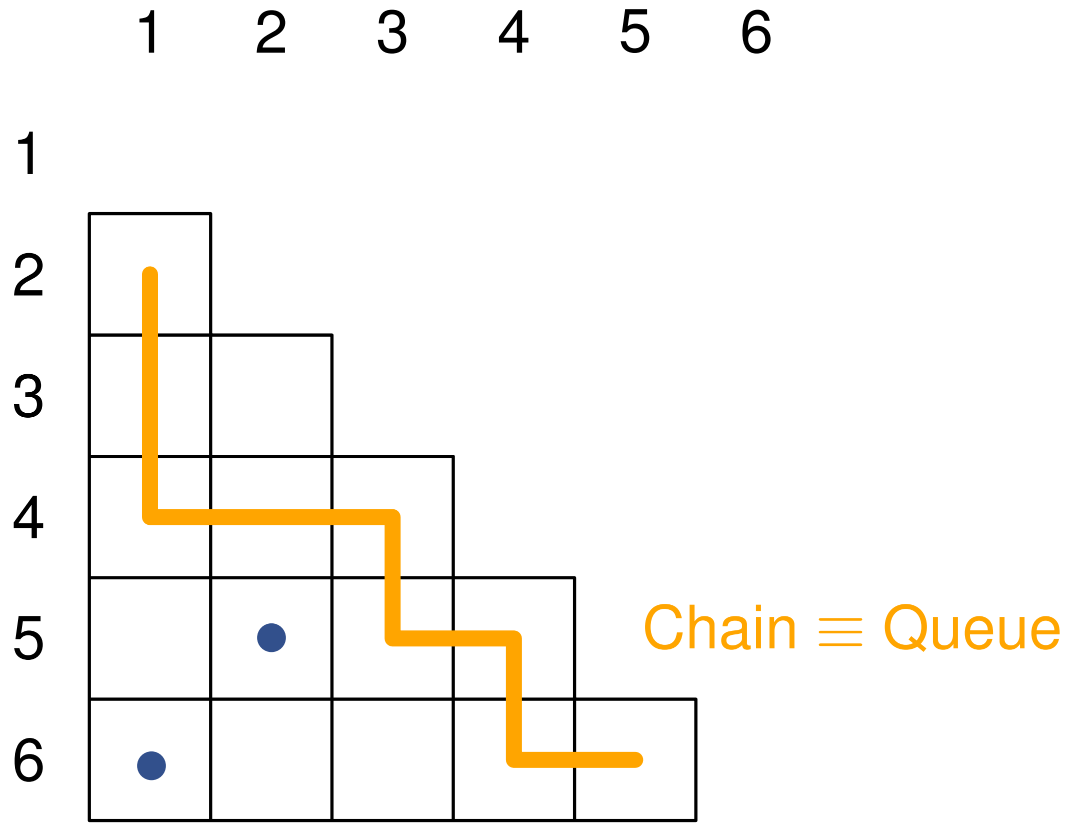
	global	⩾	union	⩾	local
Queue Number	$\lfloor n/2 \rfloor$ Heath, Rosenberg ’92				
Page Number	$\lceil n/2 \rceil$ Bernhart, Kainen ’79				

Institute of Theoretical Informatics
Algorithmics Group

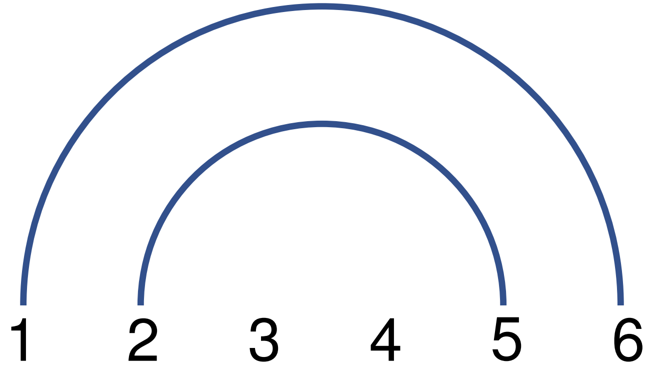
Complete Graphs

	global	⩾	union	⩾	local
Queue Number	$\lfloor n/2 \rfloor$ Heath, Rosenberg '92		$(1 - 1/\sqrt{2})n \pm O(1)$ ≈ 0.29289		
Page Number	$\lceil n/2 \rceil$ Bernhart, Kainen '79		$\geq n/3 - O(1)$ $\leq 4n/9 + O(1)$		$n/3 \pm O(1)$

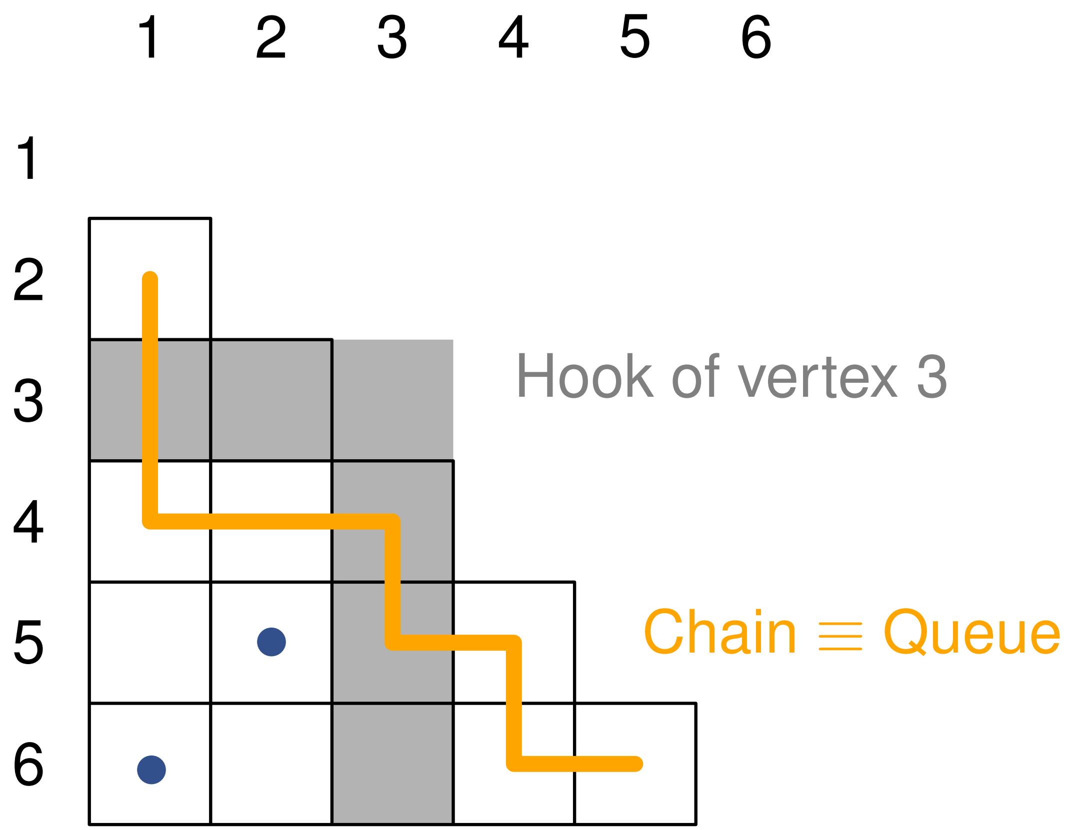
Local Queue Number



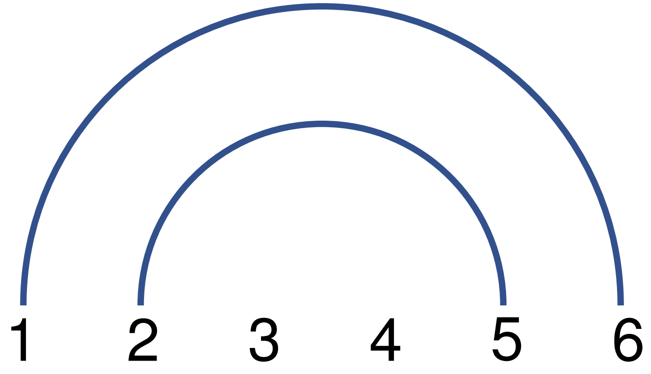
Adjacency matrix



Local Queue Number



Adjacency matrix

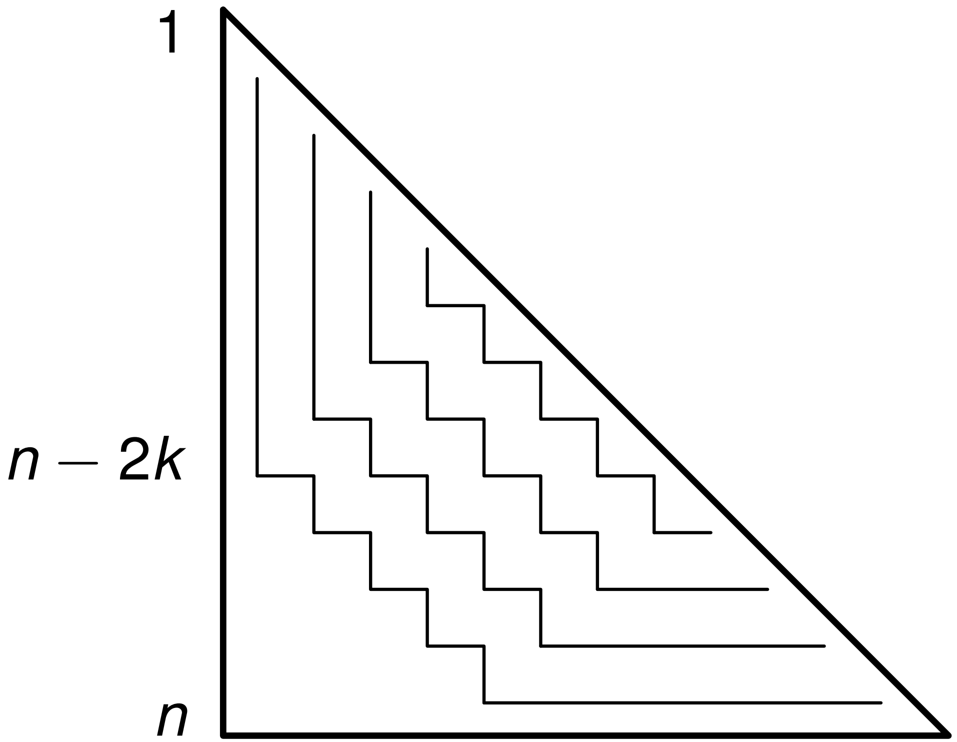


Local Queue Number $\leq k$

$\Longleftrightarrow$

each hook intersects at most k chains

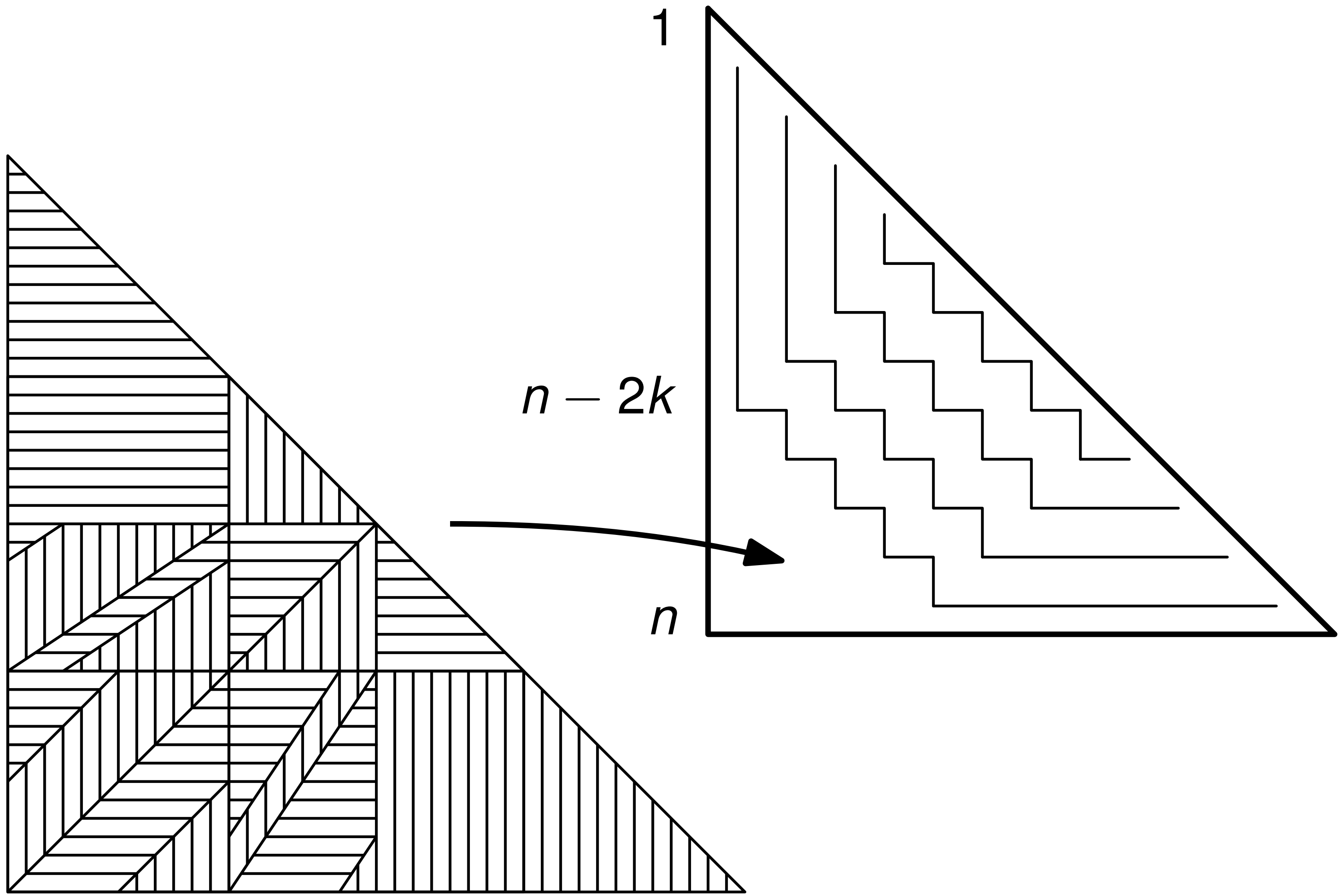
Local and Union Queue Number



$$k = (1 - 1/\sqrt{2})n$$

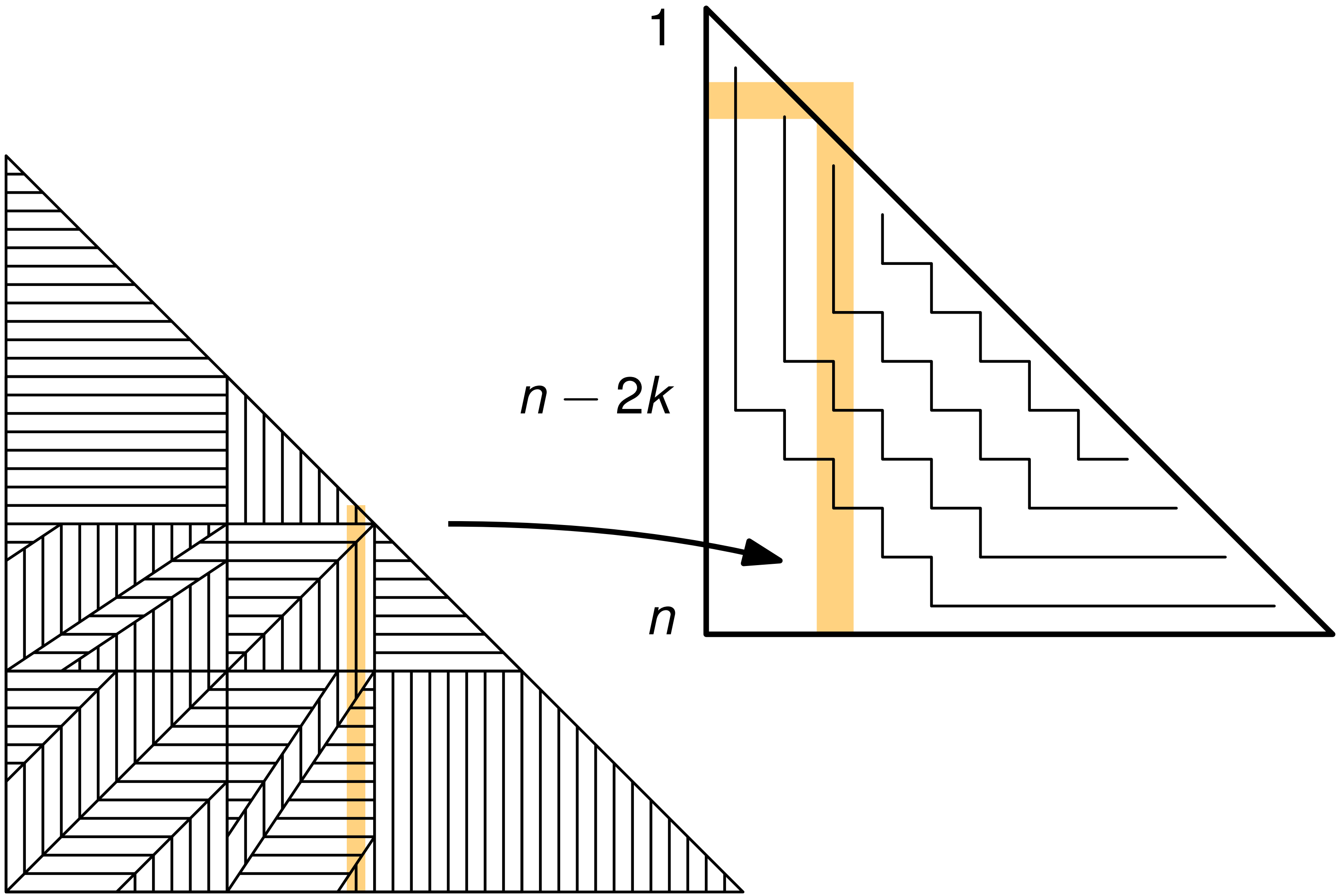
Institute of Theoretical Informatics
Algorithmics Group

Local and Union Queue Number



$k = (1 - 1/\sqrt{2})n$

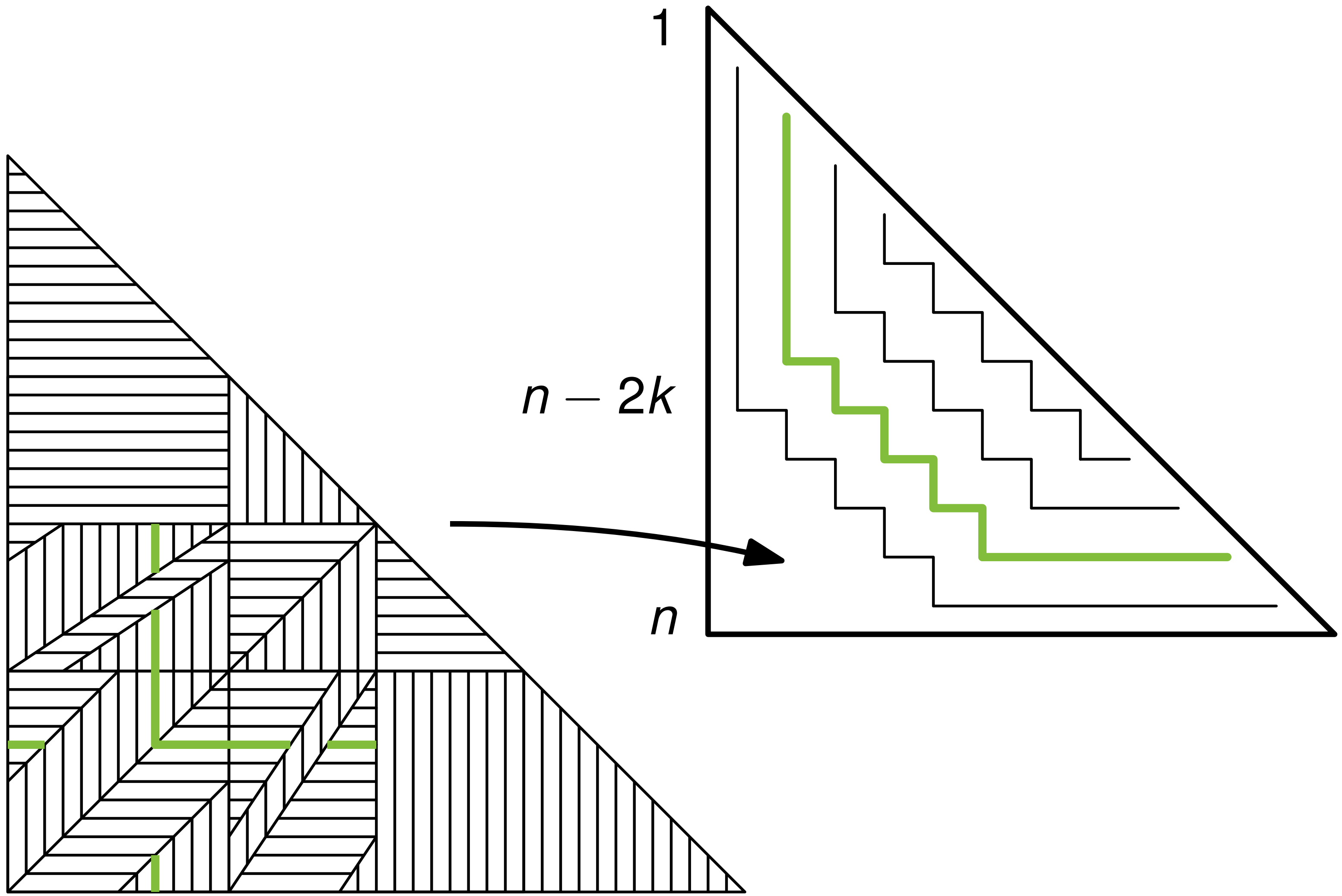
Local and Union Queue Number



$k = (1 - 1/\sqrt{2})n$

Check that each hook intersects at most $k + O(1)$ chains

Local and Union Queue Number

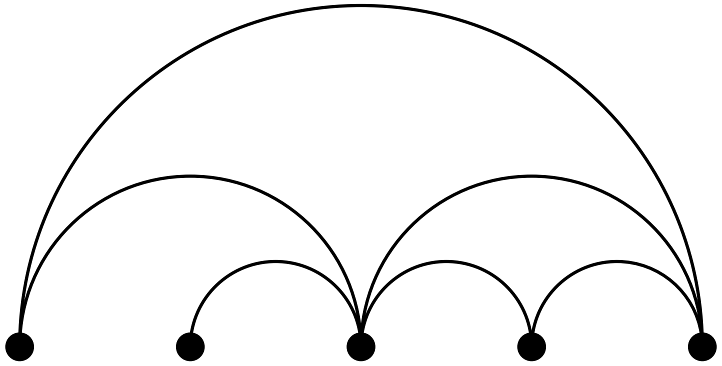


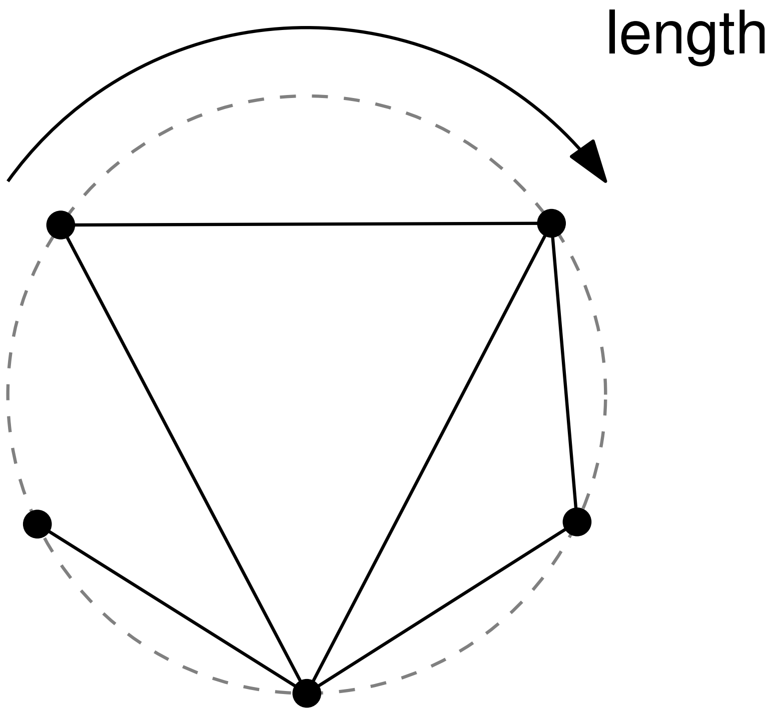
$k = (1 - 1/\sqrt{2})n$

Combine chains to $k + O(1)$ sets of chains such that

- each hook intersects at most one chain of each set
- requires to remove some edges

Local Page Number



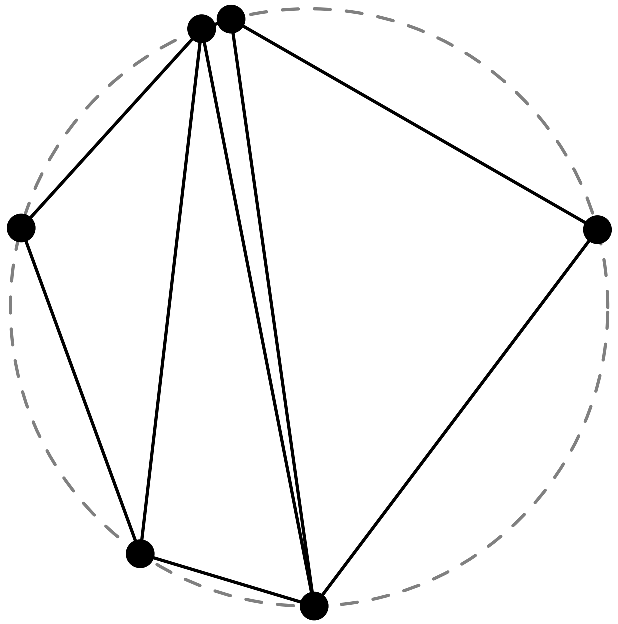


$\frac{n-1}{2}$ different edge lengths
each occurs n times in K_n

Local Page Number

Define a set of $n/18$ pages such that

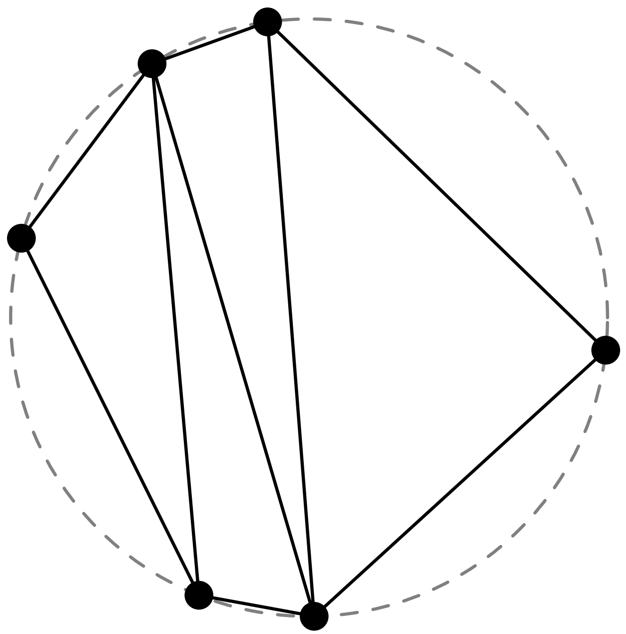
- each length is covered once
- each page contains at most six vertices



Local Page Number

Define a set of $n/18$ pages such that

- each length is covered once
- each page contains at most six vertices

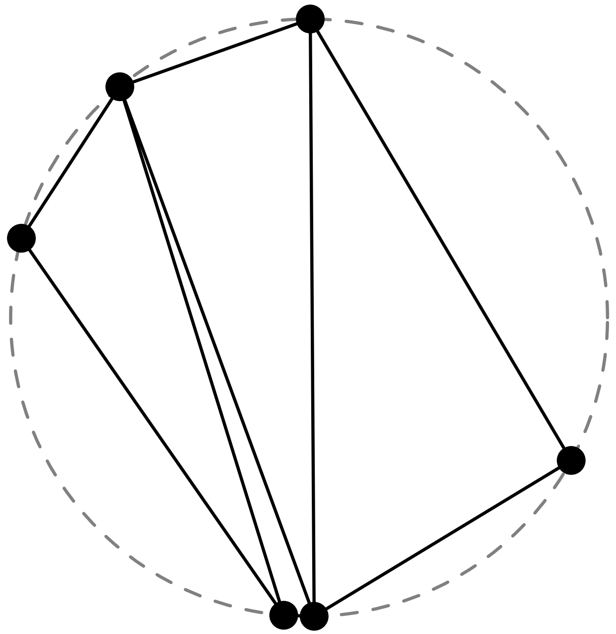


Karlsruhe Institute of Technology
Institute of Theoretical Informatics
Algorithmics Group

Local Page Number

Define a set of $n/18$ pages such that

- each length is covered once
- each page contains at most six vertices

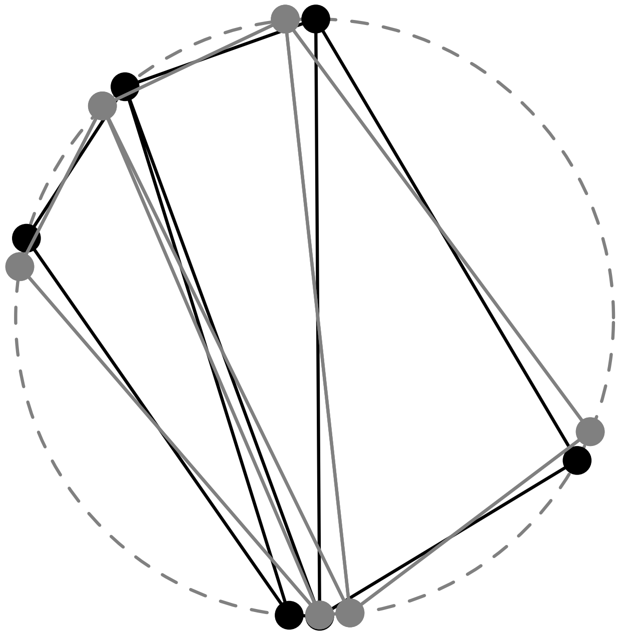


Local Page Number

Define a set of $n/18$ pages such that

- each length is covered once
- each page contains at most six vertices

Take n rotated copies of each page

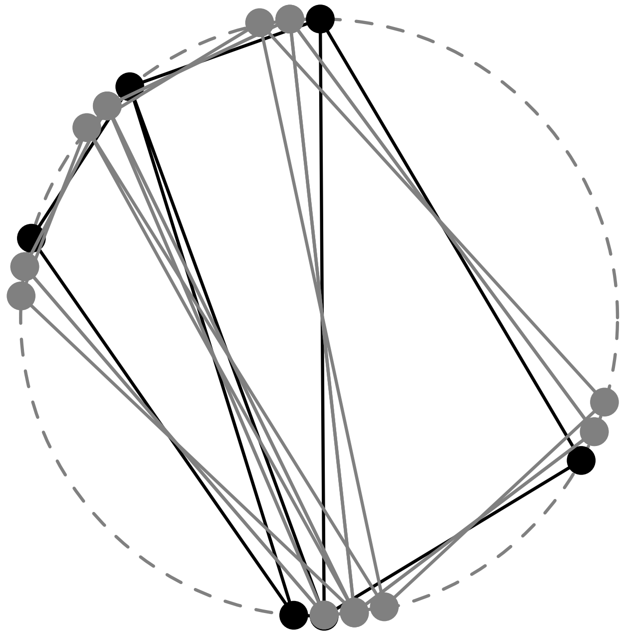


Local Page Number

Define a set of $n/18$ pages such that

- each length is covered once
- each page contains at most six vertices

Take n rotated copies of each page

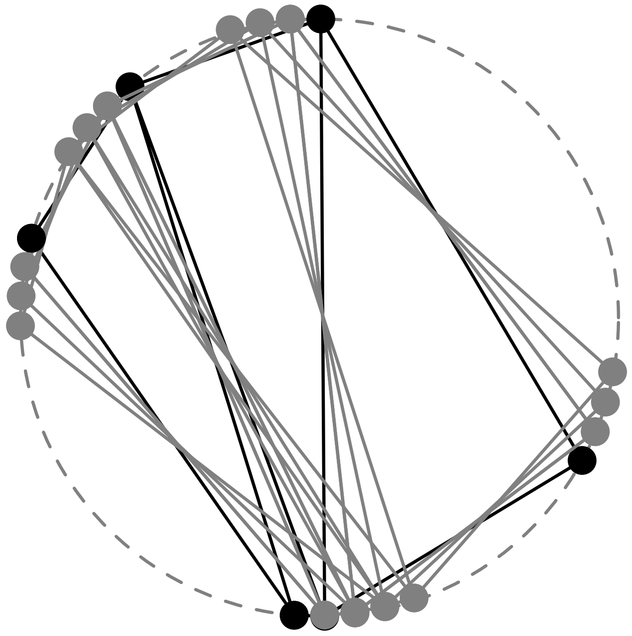


Local Page Number

Define a set of $n/18$ pages such that

- each length is covered once
- each page contains at most six vertices

Take n rotated copies of each page



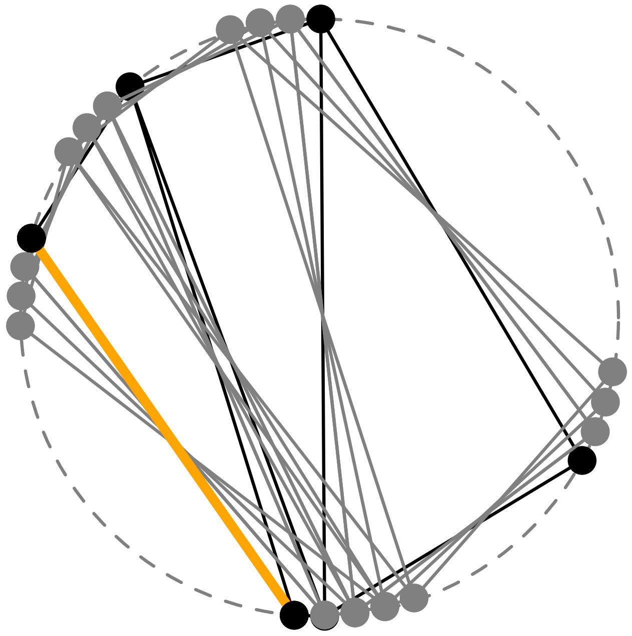
Institute of Theoretical Informatics
Algorithmics Group

Local Page Number

Define a set of $n/18$ pages such that

- each length is covered once
- each page contains at most six vertices

Take n rotated copies of each page

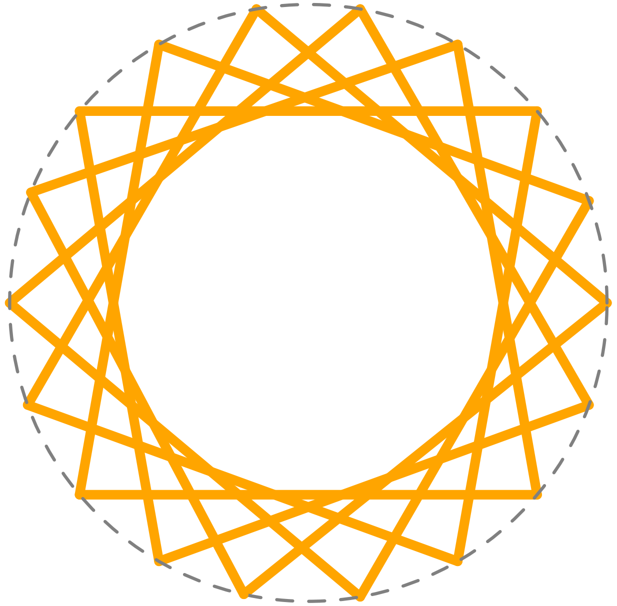


Local Page Number

Define a set of $n/18$ pages such that

- each length is covered once
- each page contains at most six vertices

Take n rotated copies of each page



- All edges are covered

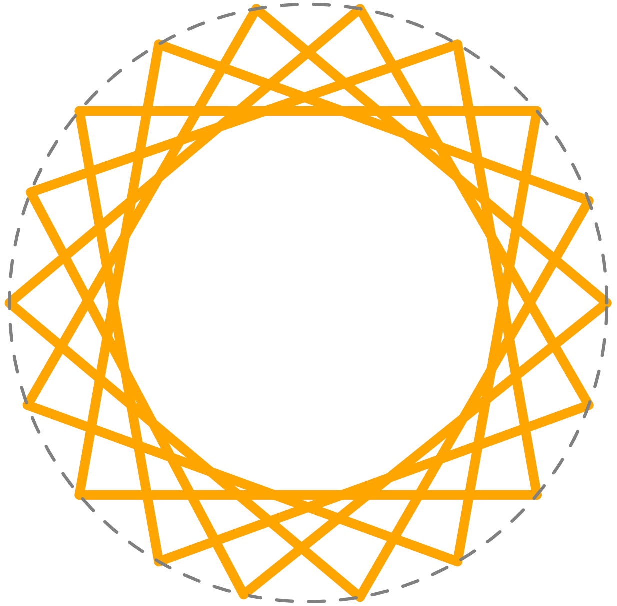
Institute of Theoretical Informatics
Algorithmics Group

Local Page Number

Define a set of $n/18$ pages such that

- each length is covered once
- each page contains at most six vertices

Take n rotated copies of each page



- All edges are covered
- # pages at each vertex:

vertices per page → # pages

$$\frac{6 \cdot n^2/18}{n} = \frac{n}{3}$$

Summary and Open Problems

Karlsruhe Institute of Technology

	global	$\geqslant$	union	$\geqslant$	local
Queue Number	$\lfloor n/2 \rfloor$ Heath, Rosenberg '92		$(1 - 1/\sqrt{2})n \pm O(1)$		
Page Number	$\lceil n/2 \rceil$ Bernhart, Kainen '79		$\geq n/3 - O(1)$ $\leq 4n/9 + O(1)$		$n/3 \pm O(1)$

Summary and Open Problems

	global $\geqslant$	union $\geqslant$	local
Queue Number	$\lfloor n/2 \rfloor$ Heath, Rosenberg '92	$(1 - 1/\sqrt{2})n \pm O(1)$	
Page Number	$\lceil n/2 \rceil$ Bernhart, Kainen '79	$\geq n/3 - O(1)$ $\leq 4n/9 + O(1)$	$n/3 \pm O(1)$

Combine pages such that each union page contains all vertices?

Institute of Theoretical Informatics
Algorithmics Group