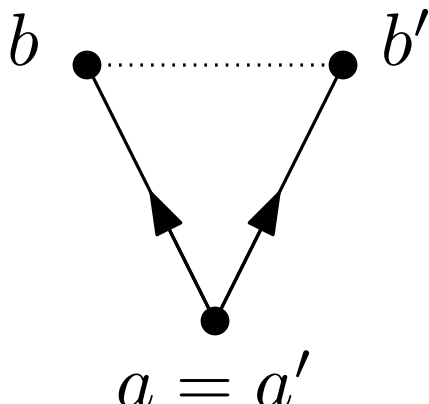
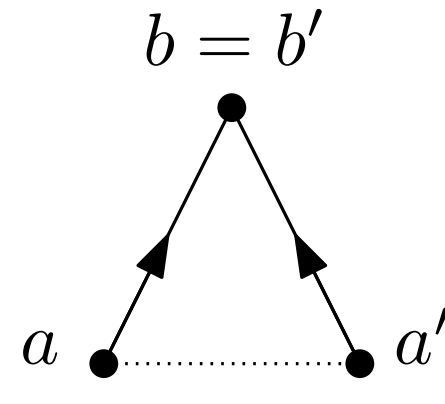


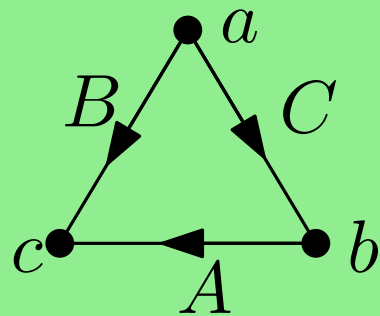
F orientation:	$ab \in F \iff ba \notin F$
F transitive:	$ab \in F \text{ and } bc \in F \implies ac \in F$
$ab \Gamma a'b'$:	  or $a = a' \text{ and } bb' \notin E \quad b = b' \text{ and } aa' \notin E$
A implication class:	$\forall ab \in A \text{ it holds } a'b' \in A \iff ab \Gamma^* a'b'$
$\mathcal{I}(G)$:	set of all implication classes
$\hat{A}$ color class:	$\{ab, ba \in E \mid ab \in A\}$
$\hat{\mathcal{I}}(G)$:	set of all color classes

Lemma 4.3 (Triangle Lemma)

$A, B, C \in \mathcal{I}(G), A \neq B, C^{-1}$

(i) $b'c' \in A \Rightarrow ab' \in C, ac' \in B$

(ii) $b'c' \in A, a'b' \in C \Rightarrow a'c' \in B$

**G-decomposition** $[B_1, \dots, B_k]$

- $E = \hat{B}_1 + \dots + \hat{B}_k$
- $B_i \in \mathcal{I}(\hat{B}_i + \dots + \hat{B}_k), i \in [k]$

rainbow triangle $\{a, b, c\}$

- $\hat{b}c \in \hat{A}, \hat{a}c \in \hat{B}, \hat{a}b \in \hat{C}$
- $\hat{A}, \hat{B}, \hat{C} \in \hat{\mathcal{I}}(G)$ pw. distinct

Thm 4.1 $A \in \mathcal{I}(G), F$ transitive
 $\Rightarrow F \cap \hat{A} = A$ or $F \cap \hat{A} = A^{-1}$

Thm 4.7

- (i) G comparability graph
 $\Leftrightarrow$ (ii) $A \cap A^{-1} = \emptyset, A \in \mathcal{I}(G)$
 $\Leftrightarrow$ (iii) $\forall$ G-decomposition $[B_1, \dots, B_k]$
 it holds $B_i \cap B_i^{-1} = \emptyset, i \in [k]$

Thm 4.4 $A \in \mathcal{I}(G)$

$\Rightarrow A = A^{-1}$ or $A \cap A^{-1} = \emptyset$
 and A, A^{-1} transitive

Thm 4.6 $A \in \mathcal{I}(G), D \in \mathcal{I}(E - \hat{A})$

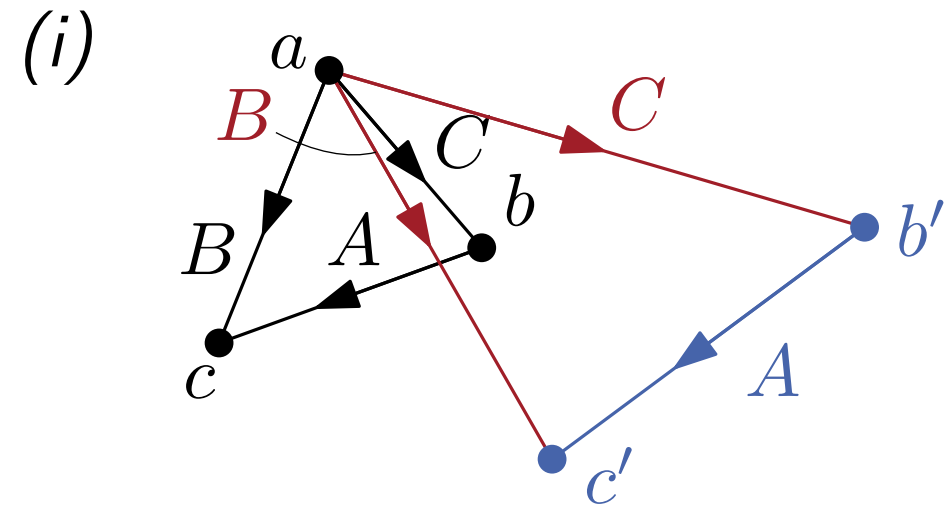
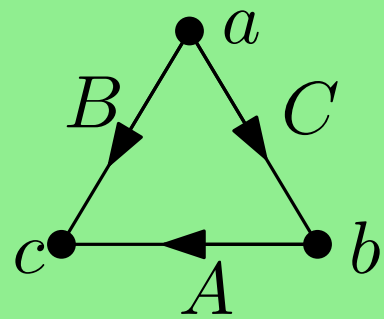
- (i) $D \in \mathcal{I}(G)$ and $A \in \mathcal{I}(E - \hat{D})$
 or (ii) $D = B + C, \hat{A}, \hat{B}, \hat{C}$ in rainbow triangle

Lemma 4.3 (Triangle Lemma)

$A, B, C \in \mathcal{I}(G), A \neq B, C^{-1}$

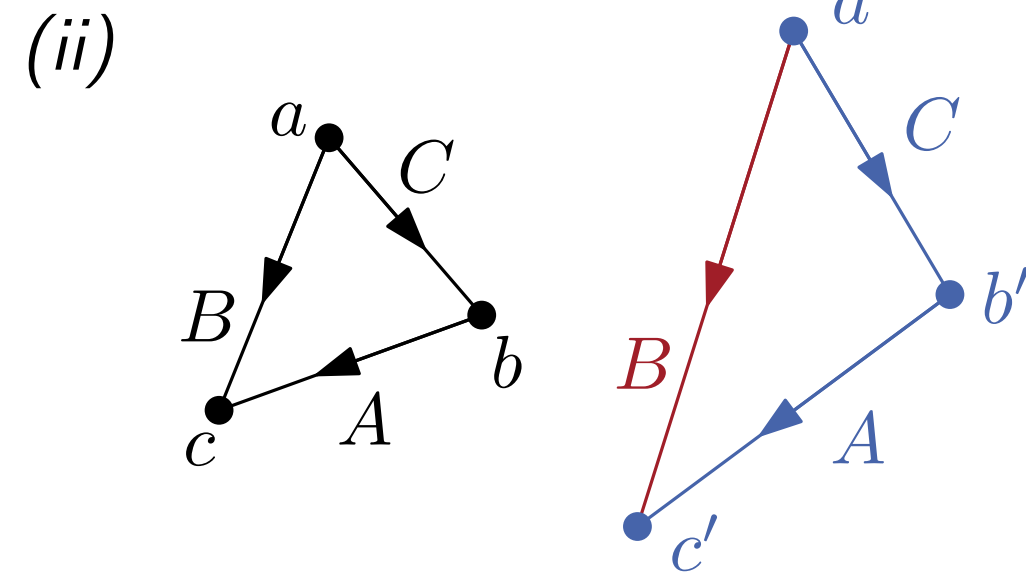
(i) $b'c' \in A \Rightarrow ab' \in C, ac' \in B$

(ii) $b'c' \in A, a'b' \in C \Rightarrow a'c' \in B$



$$b'c' \in A \Rightarrow ab' \in C, ac' \in B$$

$$A \neq B, A \neq C^{-1}$$



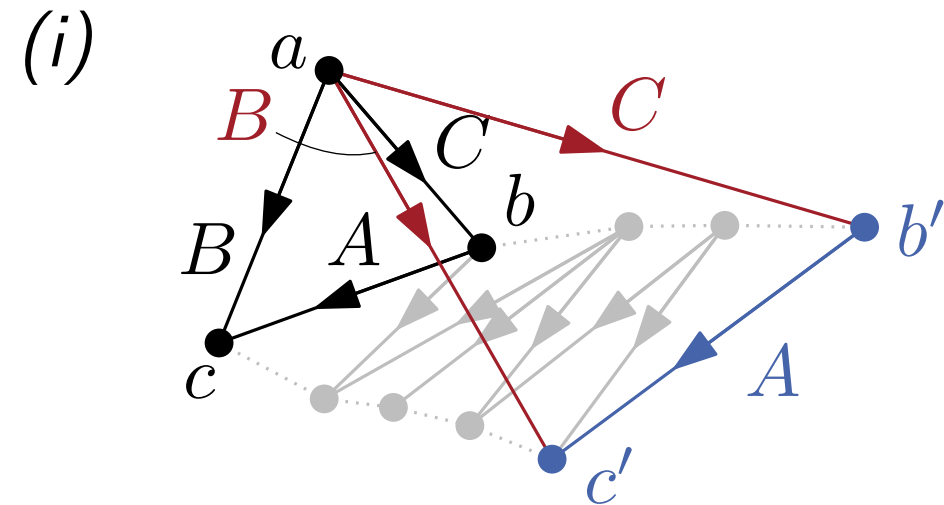
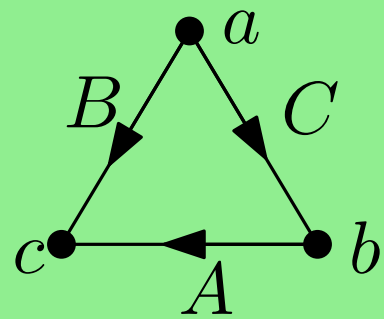
$$b'c' \in A, a'b' \in C \Rightarrow a'c' \in B$$

Lemma 4.3 (Triangle Lemma)

$A, B, C \in \mathcal{I}(G), A \neq B, C^{-1}$

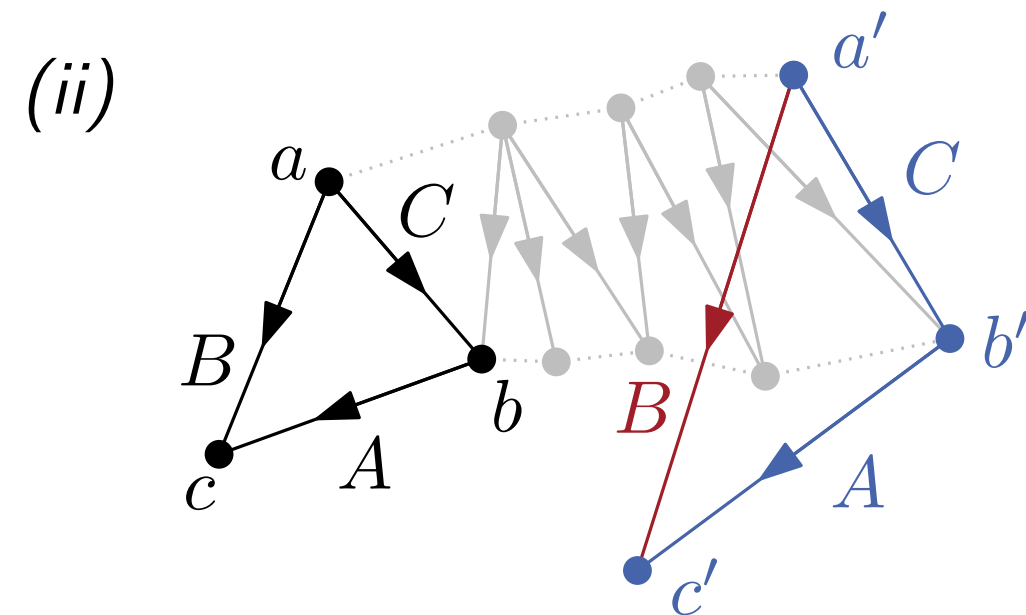
(i) $b'c' \in A \Rightarrow ab' \in C, ac' \in B$

(ii) $b'c' \in A, a'b' \in C \Rightarrow a'c' \in B$



$$b'c' \in A \Rightarrow ab' \in C, ac' \in B$$

$$A \neq B, A \neq C^{-1}$$



$$b'c' \in A, a'b' \in C \Rightarrow a'c' \in B$$

Input : undirected graph $G = (V, E)$.

Output : transitive orientation T , if it exists.

```
1   $T \leftarrow \emptyset$ ;  
2   $i \leftarrow 1$ ;  $E_i \leftarrow E$ ;  
3  while  $E_i \neq \emptyset$  do  
4      choose  $x_i y_i \in E_i$  arbitrarily;  
5      determine implication class  $B_i$  of  $E_i$  containing  $x_i y_i$ ;  
6      if  $B_i \cap B_i^{-1} \neq \emptyset$ , then  
7          return " $G$  is no comparability graph";  
8      end if  
9      add  $B_i$  to  $T$ ;  
10      $E_{i+1} \leftarrow E_i - \hat{B}_i$ ;  
11      $i \leftarrow i + 1$ ;  
12 end while  
13 return  $T$ ;
```

Algorithm 7 : Recognition of comparability graphs
