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Algorithmen zur Visualisierung von Graphen
WS 12/13
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Exercise Sheet 4

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1 Linear time construction of a Schnyder realizer.

Let G be a maximal planar graph with n vertices. Can a Schnyder labeling and a Schnyder realizer be constructed in time $O(n)$.

Hint: Find a Connection between a canonical ordering and the ordering in which the edge contraction for the construction of a Schnyder labeling is executed.

2 Property of a Schnyder realizer.

Let G be a maximal planar graph with vertices a, b, c on the outer face. Let $T_a,\ T_b,\ T_c$ be the red, the blue and the green trees of a Schnyder realizer, with sinks at vertices $a,\ b,\ c$, respectively. Let v be an internal vertex of G and denote by $P_a(v),\ P_b(v),\ P_c(v)$ the paths connecting v with $a,\ b,\ c$ in $T_a,\ T_b,\ T_c$, respectively. Show that paths $P_a(v)$, $P_b(v)$ and $P_c(v)$ do not have common vertices, except for v.

3 Induced path in a Schnyder realizer.

A path of a graph G is called *induced* if the vertices of this path are connected only by the edges of the path, i.e. path on vertices $v_1, \dots v_k$ is *induced* if for any $1 \le i, j \le n$ such that $|i-j| > 1$, edge (v_i, v_j) does not belong to G. Let G be a maximal planar graph and let T_a, T_b, T_c be a Schnyder realizer of G. Assume that the edges of $T_a,\ T_b,\ T_c$ are colored red, blue and green, respectively. Show that a directed monochromatic path in T_a, T_b, T_c is an *induced path* of G.

4 *st*-Ordering and *st*-Graphs

Let $G = (V, E)$ be a biconnected planar graph and let $f : V \to \mathbb{N}$ be the function giving an *st*-ordering of the vertices of G. In the following an undirected edge between vertices u and v is denoted by $\{u, v\}$, and an edge directed from u to v is denoted by (u, v). Let $\overrightarrow{G} = (V, \overrightarrow{E})$ be a directed graph, where $\overrightarrow{E} = \{(u, v) | \{u, v\} \in E \ \&\ f(u) < f(v)\}$. I.e. $\overrightarrow{G}$ is just an orientation of G, where each edge $\{u, v\}$ is assigned a direction from u to v if $f(u) < f(v)$ or a direction from v to u, otherwise. Prove that $\overrightarrow{G}$ is an *st*-digraph.

Hint: To achieve that prove that:

(a) $\overrightarrow{G}$ contains a single source vertex and a single sink vertex. A *source* (*sink*) of a directed graph is a vertex without incoming (outgoing) edges.
(b) $\overrightarrow{G}$ is acyclic, i.e. it does not contain any directed cycle.

5 Property of *st*-Ordering

Let $G = (V, E)$ be a biconnected planar graph with a given embedding and let $v_1, \ldots, v_n$ be an st-ordering of G such that v_1, v_n belong to the outer face of G. Let G_i denote the plane subgraph of G induced by the vertices $v_1, \ldots, v_i$. Prove that v_{i+1} belongs to the outer face of G_i.

6 Ear decomposition.

Let $G = (V, E)$ such that for each edge $\{s, t\} \in E$, G has an open ear decomposition that starts with $\{s, t\}$. Show that G is 2-connected.