Karlsruher Institut für Technologie
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Algorithmen zur Visualisierung von Graphen
WS 13/14
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Exercise Sheet 1

Assignment: October 30, 2013
Delivery: None, Discussion on November 6, 2013

1 Tree Layouts

(a) Let T be a binary tree. For each vertex v of T, we set $x(v)$ equal to the rank of v in a postorder traversal of T, and $y(v)$ equal to its depth in T.
   (i) Show that the resulting straight-line drawing is planar.
   (ii) What is the area of the drawing?
   (iii) What happens if instead of a postorder traversal we use a preorder traversal?
   (iv) Can the algorithm be extended to rooted ordered trees?

(b) Let T be a binary tree. For each vertex v of T, we set $x(v)$ equal to the rank of v in a preorder traversal of T, and $y(v)$ equal to the rank of v in a postorder traversal of T.
   (i) Show that the resulting drawing is planar and *strictly downward* (for each edge (u, v), with $depth(u) < depth(v)$, it holds that $y(u) > y(v)$).
   (ii) Show that a vertex v is in the subtree rooted at vertex u if and only if $x(v) > x(u)$ and $y(v) < y(u)$.
   (iii) Does the drawing display isomorphism of the subtrees?

2 HV-Layouts

Give an algorithm that for a given n-vertex binary tree constructs an HV-layout with minimum area in $O(n^2)$ time. Consider both ordered an non-ordered trees.

3 Outerplanar and Series-Parallel Graphs

A graph G is called *outerplanar* if it has a planar drawing where all vertices lie on the boundary of the external face. Show that every biconnected outerplanar graph is series-parallel.

4 Visibility Representation

In a *visibility representation* of a graph $G = (V, E)$ the vertices are represented by horizontal segments. We say that two vertices u and v *see* each other, if they can be connected by a vertical rectangle of non-zero widch that does not cross any other vertex-segment. Thus, in a visibility representation of G, two vertices u, v see each other iff $(u, v) \in E$. The bottom figure on the left shows a visibility representation of the graph on top.
Show that each series-parallel graph has a visibility representation.