Karlsruhe Institute of Technology

Algorithmische Graphentheorie
Übung 3

Übung 3 · 27. November 2014
Thomas Bläsius

Institute of Theoretical Informatics · Algorithmics Group

$$
\begin{pmatrix}
1&1&1&0&0&0&0&0&0&0&0&0&0\\
0&1&1&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&1&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&1&1\\
0&0&1&1&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&1&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&1&1&0&0\\
1&0&0&0&0&0&0&0&0&0&0&1&1\\
0&0&0&1&1&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&1&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&1&1&0\\
1&1&0&0&0&0&0&0&0&0&0&0&1
\end{pmatrix}
\cdot
\begin{pmatrix}
0&0&0&1&0&0&1&0&0&1&0&0&1\\
1&0&0&0&1&0&0&1&0&0&1&0&0\\
1&0&0&1&0&0&0&1&0&0&1&0&0\\
1&0&0&1&0&0&1&0&0&0&1&0&0\\
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0&0&1&0&0&1&0&0&1&0&0&0&1\\
0&0&1&0&0&1&0&0&1&0&0&1&0
\end{pmatrix}^T
=
\begin{pmatrix}
0&1&1&1&1&1&1&1&1&1&1&1&1\\
1&0&1&1&1&1&1&1&1&1&1&1&1\\
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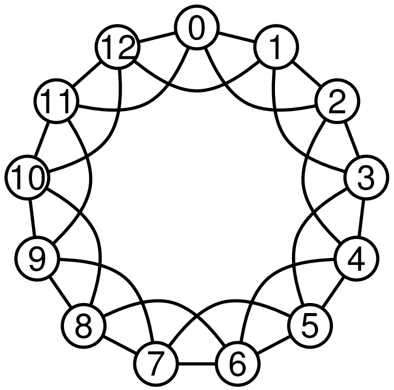
1&1&1&0&1&1&1&1&1&1&1&1&1\\
1&1&1&1&0&1&1&1&1&1&1&1&1\\
1&1&1&1&1&0&1&1&1&1&1&1&1\\
1&1&1&1&1&1&0&1&1&1&1&1&1\\
1&1&1&1&1&1&1&0&1&1&1&1&1\\
1&1&1&1&1&1&1&1&0&1&1&1&1\\
1&1&1&1&1&1&1&1&1&0&1&1&1\\
1&1&1&1&1&1&1&1&1&1&0&1&1\\
1&1&1&1&1&1&1&1&1&1&1&0&1\\
1&1&1&1&1&1&1&1&1&1&1&1&0
\end{pmatrix}
$$

Partitionierbare Graphen – C_{13}^2



Partitionierbare Graphen – C_{13}^2

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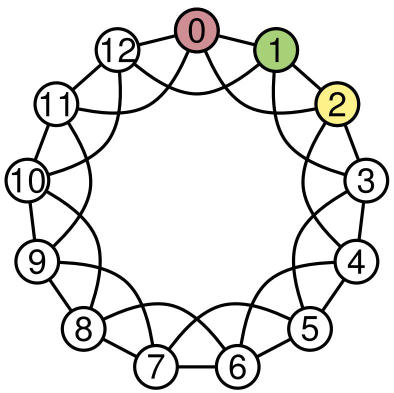

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0

- wähle maximale Clique $\{0, 1, 2\}$

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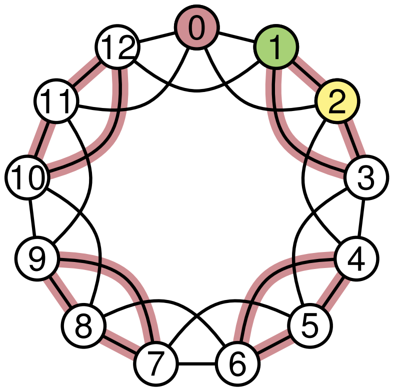
Partitionierbare Graphen – C_{13}^2



0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1

- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0

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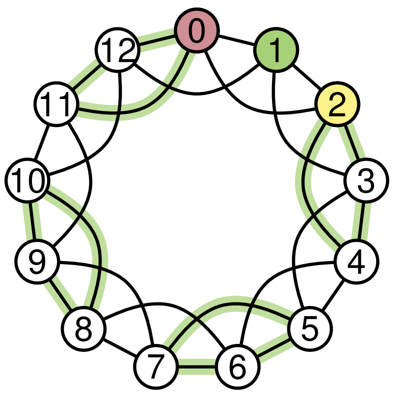
Partitionierbare Graphen – C_{13}^2



0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1

- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1

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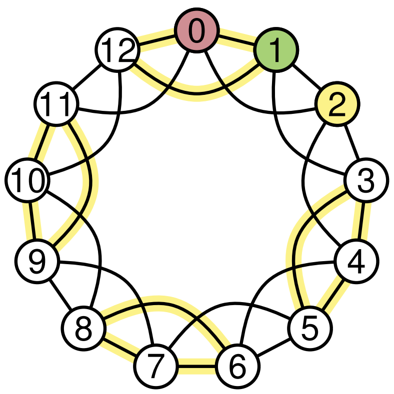
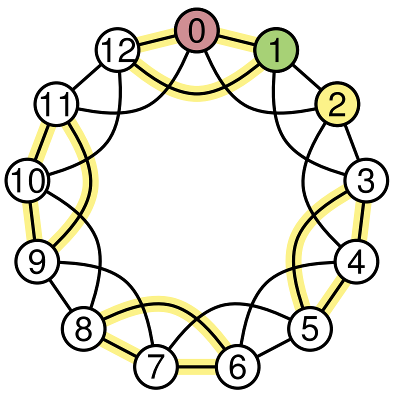
Partitionierbare Graphen – C_{13}^2



0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1

- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2

Partitionierbare Graphen – C_{13}^2



$A =$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1

- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2

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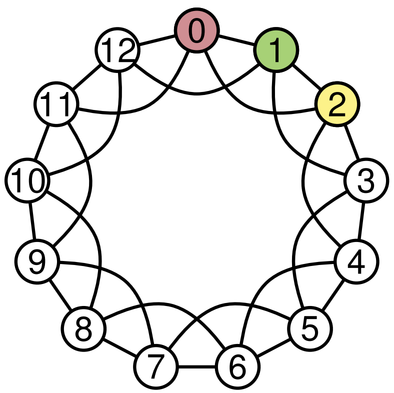
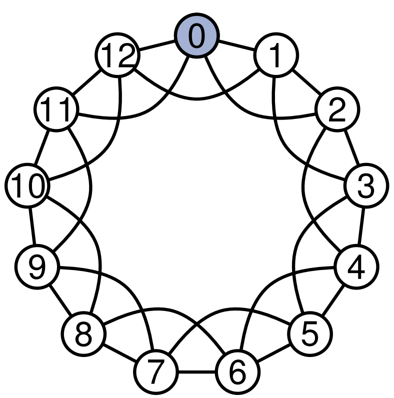
Partitionierbare Graphen – C_{13}^2



$A =$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1



- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2
- lösche einen Knoten v aus Clique K_i

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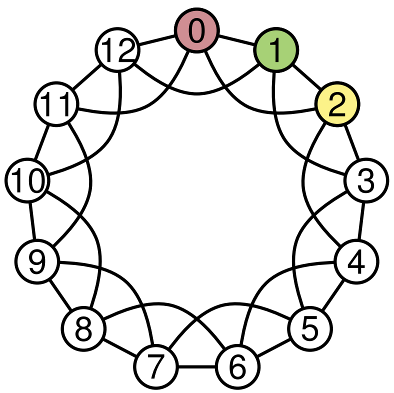
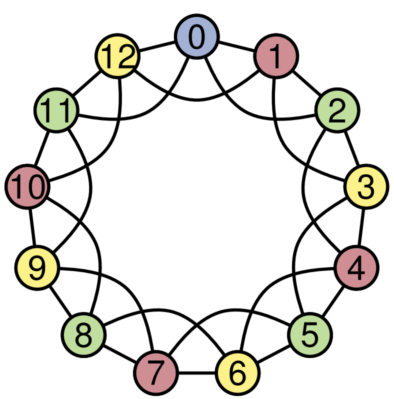
Partitionierbare Graphen – C_{13}^2



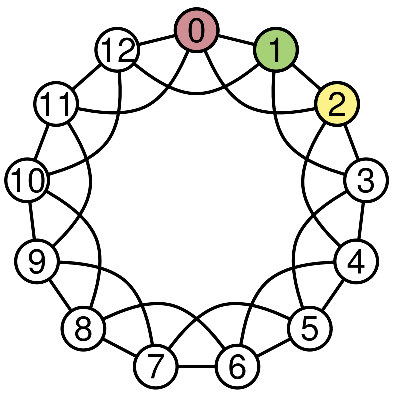
$A =$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1



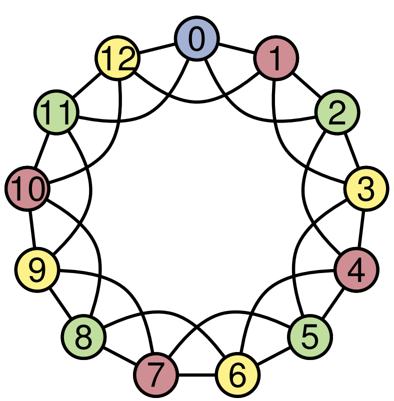
- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2
- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v

Partitionierbare Graphen – C_{13}^2



$A =$

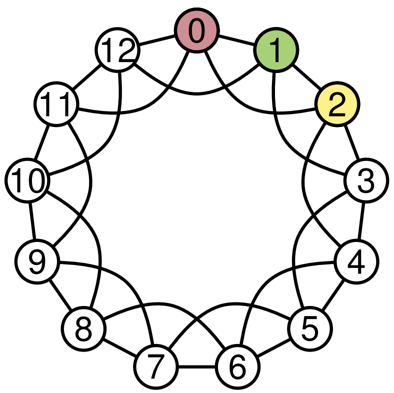
0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1



0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1

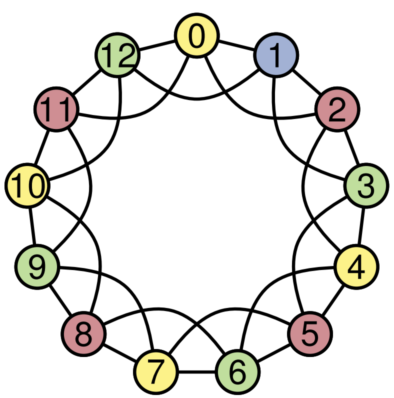
- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2
- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v
- wähle unabhängige Menge die keinen Knoten aus K_i enthält

Partitionierbare Graphen – C_{13}^2



$A =$

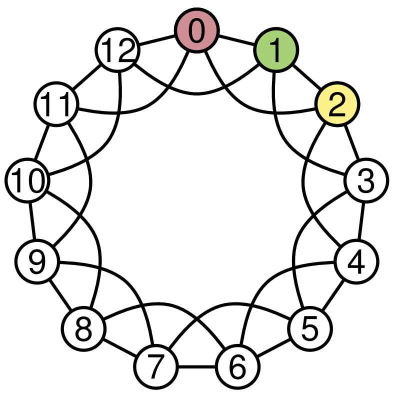
0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1



0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	1	0	0	1	0	0

- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2
- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v
- wähle unabhängige Menge die keinen Knoten aus K_i enthält

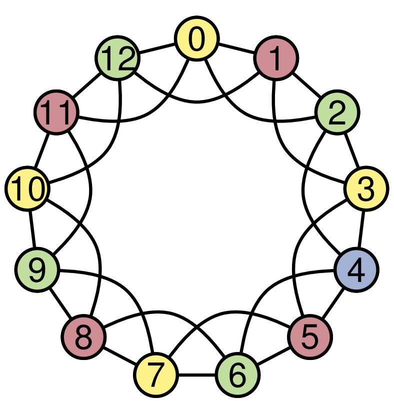
Partitionierbare Graphen – C_{13}^2



$A =$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1

- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2



0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	1	0	0	1	0	0

- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v
- wähle unabhängige Menge die keinen Knoten aus K_i enthält

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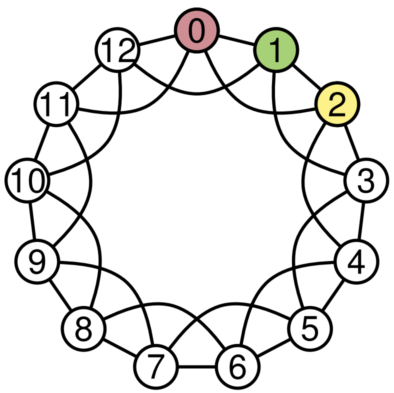
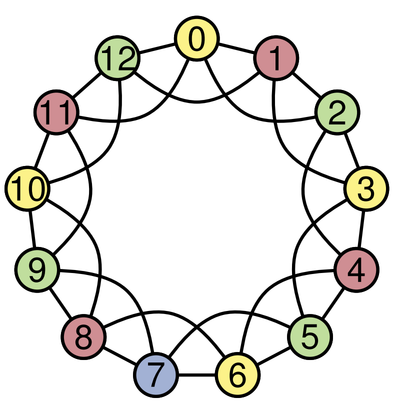
Partitionierbare Graphen – C_{13}^2



$A =$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1



0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	1	0	0	0	1	0	0

- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2
- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v
- wähle unabhängige Menge die keinen Knoten aus K_i enthält

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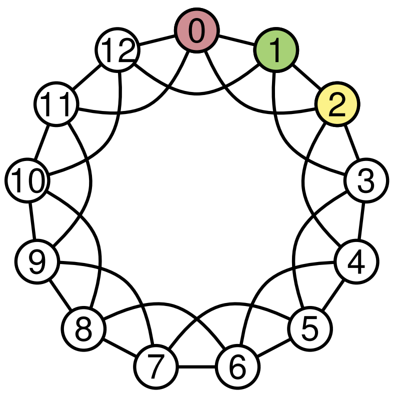
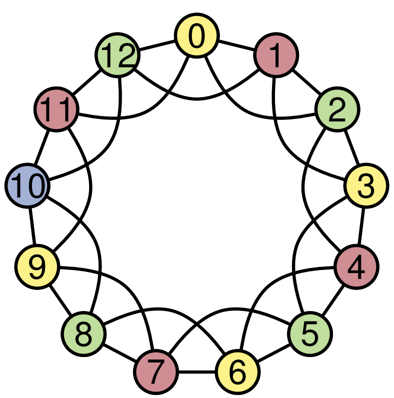
Partitionierbare Graphen – C_{13}^2



$A =$

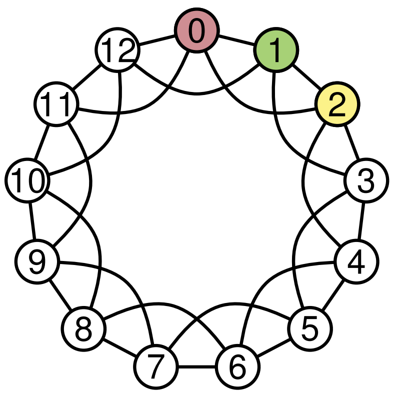
0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1



0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	1	0	0	0	1	0	0
1	0	0	1	0	0	1	0	0	1	0	0	0

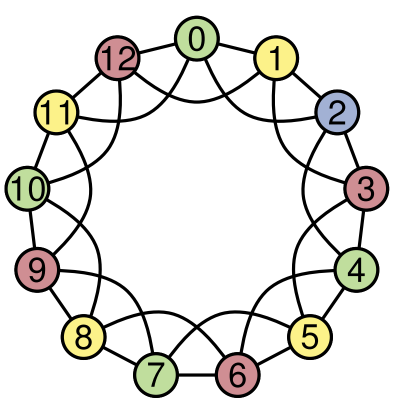
- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2
- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v
- wähle unabhängige Menge die keinen Knoten aus K_i enthält

Partitionierbare Graphen – C_{13}^2



$A =$

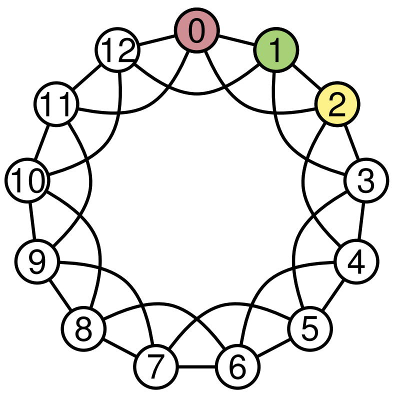
0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1



0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	1	0	0	0	1	0	0
1	0	0	1	0	0	1	0	0	1	0	0	0
0	1	0	0	0	1	0	0	1	0	0	1	0

- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2
- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v
- wähle unabhängige Menge die keinen Knoten aus K_i enthält

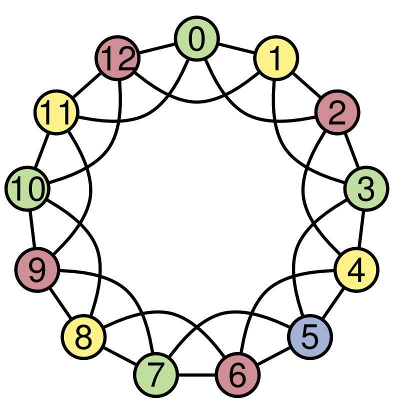
Partitionierbare Graphen – C_{13}^2



$A =$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1

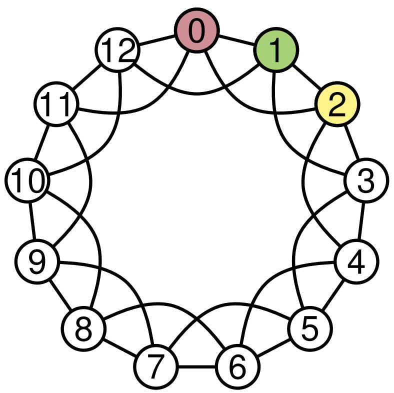
- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2



0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	1	0	0	0	1	0	0
1	0	0	1	0	0	1	0	0	1	0	0	0
0	1	0	0	0	1	0	0	1	0	0	1	0
0	1	0	0	1	0	0	0	1	0	0	1	0

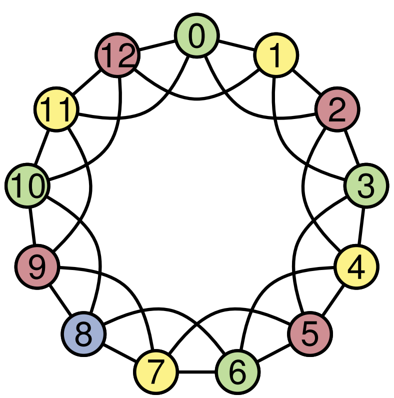
- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v
- wähle unabhängige Menge die keinen Knoten aus K_i enthält

Partitionierbare Graphen – C_{13}^2



$A =$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1

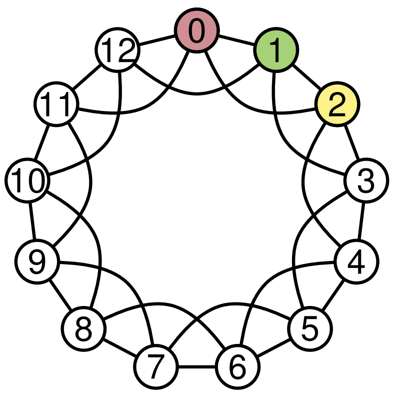


0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	1	0	0	0	1	0	0
1	0	0	1	0	0	1	0	0	1	0	0	0
0	1	0	0	0	1	0	0	1	0	0	1	0
0	1	0	0	1	0	0	0	1	0	0	1	0
0	1	0	0	1	0	0	1	0	0	0	1	0

- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2

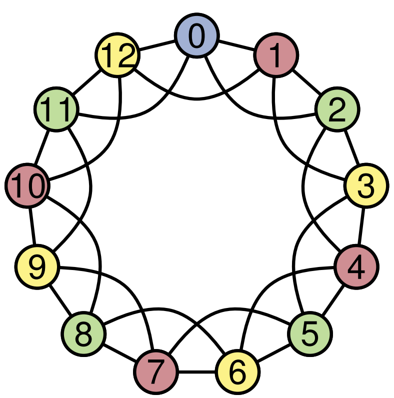
- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v
- wähle unabhängige Menge die keinen Knoten aus K_i enthält

Partitionierbare Graphen – C_{13}^2



$A =$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1

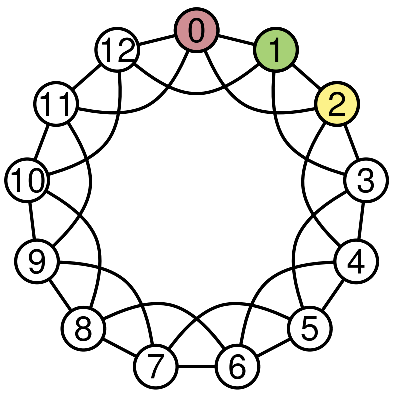


0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	1	0	0	0	1	0	0
1	0	0	1	0	0	1	0	0	1	0	0	0
0	1	0	0	0	1	0	0	1	0	0	1	0
0	1	0	0	1	0	0	0	1	0	0	1	0
0	1	0	0	1	0	0	1	0	0	0	1	0
0	1	0	0	1	0	0	1	0	0	1	0	0

- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2

- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v
- wähle unabhängige Menge die keinen Knoten aus K_i enthält

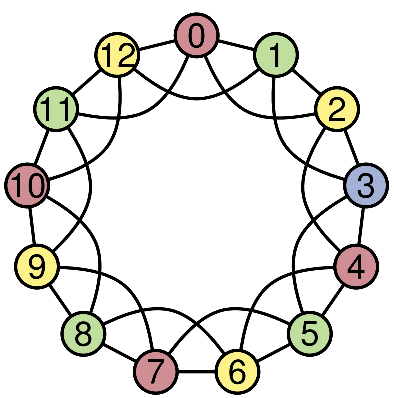
Partitionierbare Graphen – C_{13}^2



$A =$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1

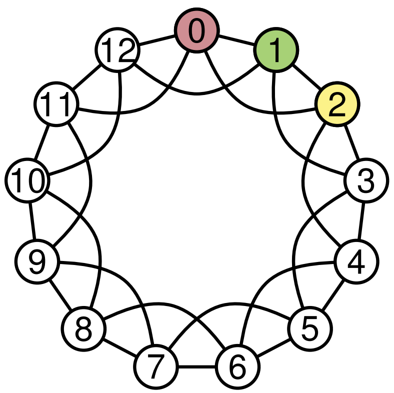
- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2



0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	1	0	0	0	1	0	0
1	0	0	1	0	0	1	0	0	1	0	0	0
0	1	0	0	0	1	0	0	1	0	0	1	0
0	1	0	0	1	0	0	0	1	0	0	1	0
0	1	0	0	1	0	0	1	0	0	0	1	0
0	1	0	0	1	0	0	1	0	0	1	0	0
0	0	1	0	0	0	1	0	0	1	0	0	1

- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v
- wähle unabhängige Menge die keinen Knoten aus K_i enthält

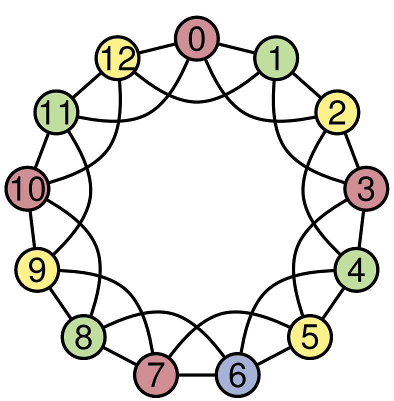
Partitionierbare Graphen – C_{13}^2



$A =$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1

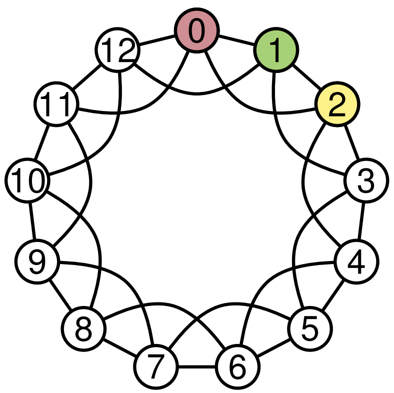
- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2



0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	1	0	0	0	1	0	0
1	0	0	1	0	0	1	0	0	1	0	0	0
0	1	0	0	0	1	0	0	1	0	0	1	0
0	1	0	0	1	0	0	0	1	0	0	1	0
0	1	0	0	1	0	0	1	0	0	0	1	0
0	1	0	0	1	0	0	1	0	0	1	0	0
0	0	1	0	0	0	1	0	0	1	0	0	1
0	0	1	0	0	1	0	0	0	1	0	0	1

- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v
- wähle unabhängige Menge die keinen Knoten aus K_i enthält

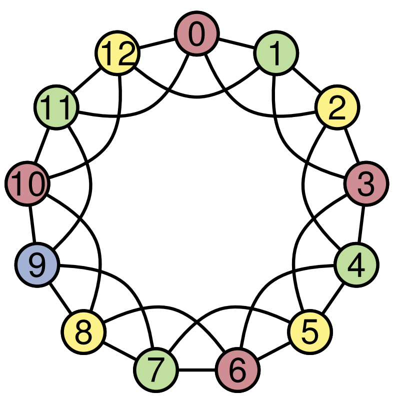
Partitionierbare Graphen – C_{13}^2



$A =$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1

- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2



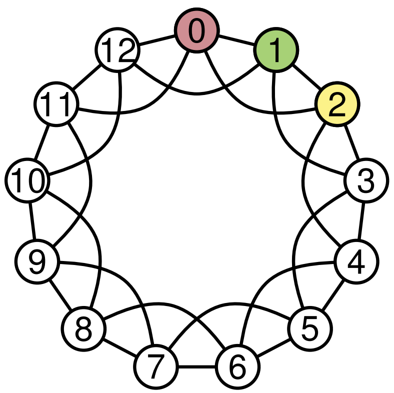
0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	1	0	0	0	1	0	0
1	0	0	1	0	0	1	0	0	1	0	0	0
0	1	0	0	0	1	0	0	1	0	0	1	0
0	1	0	0	1	0	0	0	1	0	0	1	0
0	1	0	0	1	0	0	1	0	0	0	1	0
0	1	0	0	1	0	0	1	0	0	1	0	0
0	0	1	0	0	0	1	0	0	1	0	0	1
0	0	1	0	0	1	0	0	0	1	0	0	1
0	0	1	0	0	1	0	0	1	0	0	0	1

- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v
- wähle unabhängige Menge die keinen Knoten aus K_i enthält

Institute of Theoretical Informatics
Algorithmics Group

Partitionierbare Graphen – C_{13}^2

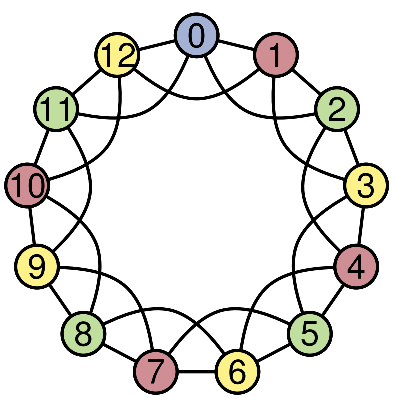
Karlsruhe Institute of Technology



$A =$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1

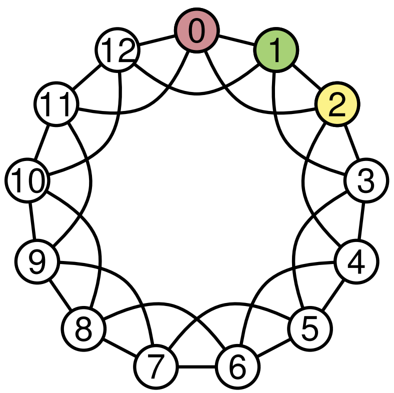
- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2



0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	1	0	0	0	1	0	0
1	0	0	1	0	0	1	0	0	1	0	0	0
0	1	0	0	0	1	0	0	1	0	0	1	0
0	1	0	0	1	0	0	0	1	0	0	1	0
0	1	0	0	1	0	0	1	0	0	0	1	0
0	1	0	0	1	0	0	1	0	0	1	0	0
0	0	1	0	0	0	1	0	0	1	0	0	1
0	0	1	0	0	1	0	0	0	1	0	0	1
0	0	1	0	0	1	0	0	1	0	0	0	1
0	0	1	0	0	1	0	0	1	0	0	1	0

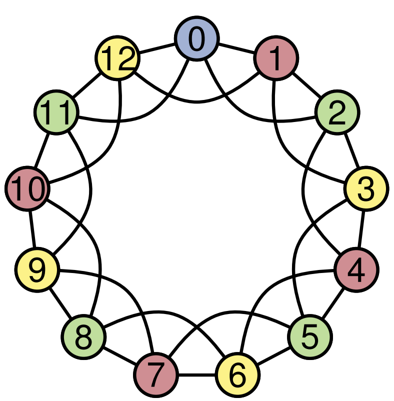
- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v
- wähle unabhängige Menge die keinen Knoten aus K_i enthält

Partitionierbare Graphen – C_{13}^2



$A =$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1



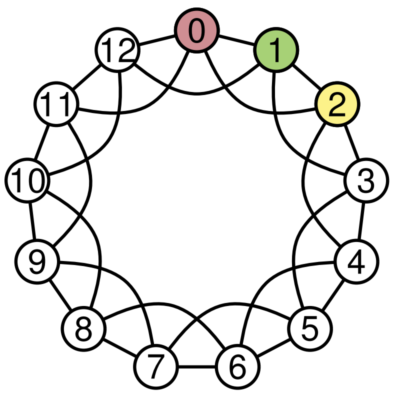
$B =$

0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	1	0	0	0	1	0	0
1	0	0	1	0	0	1	0	0	1	0	0	0
0	1	0	0	0	1	0	0	1	0	0	1	0
0	1	0	0	1	0	0	0	1	0	0	1	0
0	1	0	0	1	0	0	1	0	0	0	1	0
0	1	0	0	1	0	0	1	0	0	1	0	0
0	0	1	0	0	0	1	0	0	1	0	0	1
0	0	1	0	0	1	0	0	0	1	0	0	1
0	0	1	0	0	1	0	0	1	0	0	0	1
0	0	1	0	0	1	0	0	1	0	0	1	0

- wähle maximale Clique $\{0, 1, 2\}$
- wähle minimales Clique-Cover ohne Knoten 0
- wähle minimales Clique-Cover ohne Knoten 1
- wähle minimales Clique-Cover ohne Knoten 2

- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v
- wähle unabhängige Menge die keinen Knoten aus K_i enthält

Partitionierbare Graphen – C_{13}^2



$A =$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1

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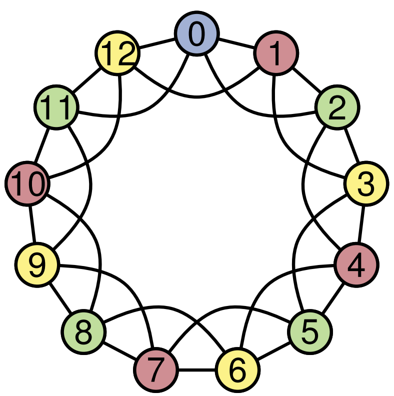

$B =$

0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	1	0	0	0	1	0	0
1	0	0	1	0	0	1	0	0	1	0	0	0
0	1	0	0	0	1	0	0	1	0	0	1	0
0	1	0	0	1	0	0	0	1	0	0	1	0
0	1	0	0	1	0	0	1	0	0	0	1	0
0	1	0	0	1	0	0	1	0	0	1	0	0
0	0	1	0	0	0	1	0	0	1	0	0	1
0	0	1	0	0	1	0	0	0	1	0	0	1
0	0	1	0	0	1	0	0	1	0	0	0	1
0	0	1	0	0	1	0	0	1	0	0	1	0

- lösche einen Knoten v aus Clique K_i
- wähle minimale Färbung ohne Knoten v
- wähle unabhängige Menge die keinen Knoten aus K_i enthält

Eigenschaften von A und B

- Zeilen-/Spaltensumme von A ist ω (= 3)
- Zeilen-/Spaltensumme von B ist α (= 4)
- $AB^T = J - I = \begin{pmatrix} 0 & & 1 \\ & \ddots & \\ 1 & & 0 \end{pmatrix}$

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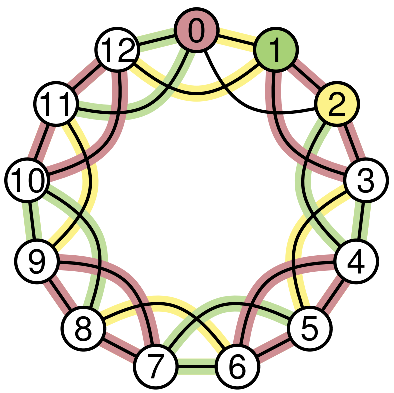

$A =$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0
1	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	0	0	0	0	1

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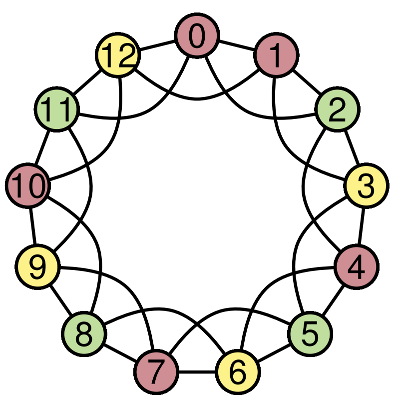

$B =$

0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	1	0	0	1	0	0	1	0	0	1
1	0	0	0	1	0	0	1	0	0	1	0	0
1	0	0	1	0	0	0	1	0	0	1	0	0
1	0	0	1	0	0	1	0	0	0	1	0	0
1	0	0	1	0	0	1	0	0	1	0	0	0
0	1	0	0	0	1	0	0	1	0	0	1	0
0	1	0	0	1	0	0	0	1	0	0	1	0
0	1	0	0	1	0	0	1	0	0	0	1	0
0	1	0	0	1	0	0	1	0	0	1	0	0
0	0	1	0	0	0	1	0	0	1	0	0	1
0	0	1	0	0	1	0	0	0	1	0	0	1
0	0	1	0	0	1	0	0	1	0	0	0	1
0	0	1	0	0	1	0	0	1	0	0	1	0

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