Exercise Sheet 3

Assignment: November 30, 2016
Discussion: December 14, 2016

1 Linear time construction of a Schnyder realizer.

Let G be a maximal planar graph with n vertices. Can a Schnyder labeling and a Schnyder realizer be constructed in time $O(n)$?

Hint: Find a connection between a canonical ordering and the ordering in which the edge contraction for the construction of a Schnyder labeling is executed.

2 Induced path in a Schnyder realizer.

A path of a graph G is called *induced* if the vertices of this path are connected only by the edges of the path, i.e. path on vertices $v_1, \dots v_k$ is *induced* if for any $1 \leq i, j \leq n$ such that $|i - j| > 1$, edge (v_i, v_j) does not belong to G. Let G be a maximal planar graph and let T_a, T_b, T_c be a Schnyder realizer of G. Assume that the edges of T_a, T_b, T_c are colored red, blue and green, respectively. Show that a directed monochromatic path in T_a, T_b, T_c is an *induced path* of G.

3 Forces

In the following we consider force-based layout methods.

(a) Define forces that ensure that

  (i) vertices closely stay to pre-defined positions,
  (ii) vertices are closely placed to the x-axis,
  (iii) edges are aligned parallel to the y-axis,

(b) Let $G = (V, E)$ be a graph and assume that we are given a clustering $\mathcal{C}$ for G, i.e., a partition of V in pairwise disjoint subsets $C_1 \dots, C_k$ with $\cup_{C \in \mathcal{C}} C = V$.

Define forces that make sure that vertices of the same cluster lie closely together, while vertices of different clusters lie apart from each other.

(c) For a vertex u with position $p_u = (x_u, y_u)$ we define the translation vector of u by disp : $\mathbb{R}^2 \to \mathbb{R}^2$ where

$$\operatorname{disp}(p_u) = \sum_{\{u,v\}\in E} \frac{||p_v - p_u||^2}{d_{uv}}(p_v - p_u) - \sum_{v\in V} \frac{C}{||p_v - p_u||^2}(p_v - p_u).$$

Hereby, $C \in \mathbb{R}$ and $d_{uv} \in \mathbb{R}$ (for all edges $\{u, v\} \in E$) are constants.

Determine the potential function pot : $\mathbb{R}^2 \to \mathbb{R}$ such that $\operatorname{disp}(p_u) = -\nabla \operatorname{pot}(p_u)$, i.e., the translation vector of u is equal to the negative gradient of the potential function.

4 Stability of Springembedder

Let $G = (V, E)$ be a graph with $V = \{a, b, c, d\}$ and $E = \{\{a, b\}, \{a, c\}, \{a, d\}\}$.

Find a stable output for G produced by the spring-embedder algorithm by Fruchterman and Reingold.

5 Vertices with Area > 0

So far we have assumed for the spring-embedder algorithm that vertices are represented by points. How can the approach be adapted if the vertices are represented by circles that should not overlap? What about rectangles?