# Algorithms for Graph Visualization

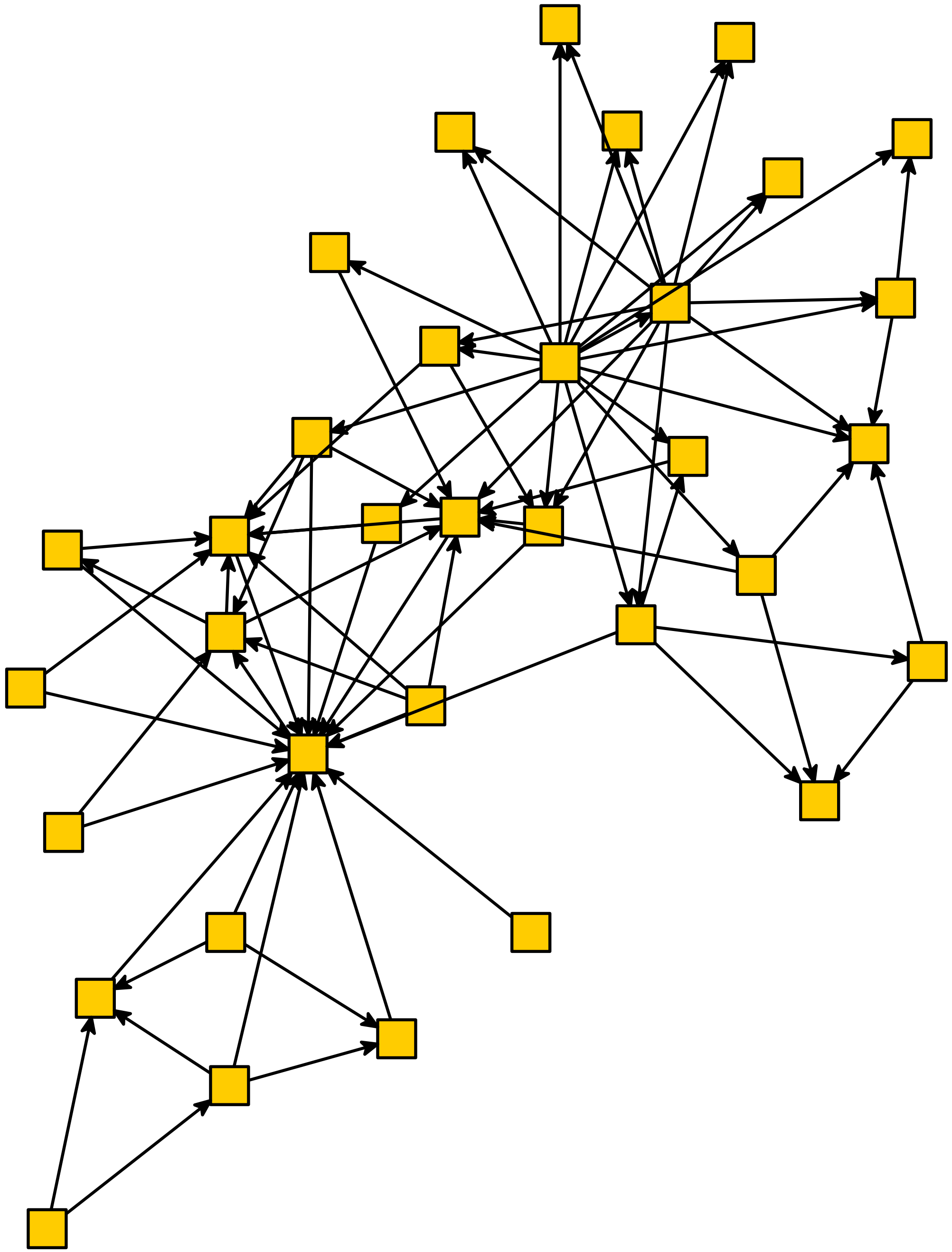
## Layered Layout

INSTITUT FÜR THEORETISCHE INFORMATIK · FAKULTÄT FÜR INFORMATIK

**Tamara Mchedlidze**

4.12.2018

# Example



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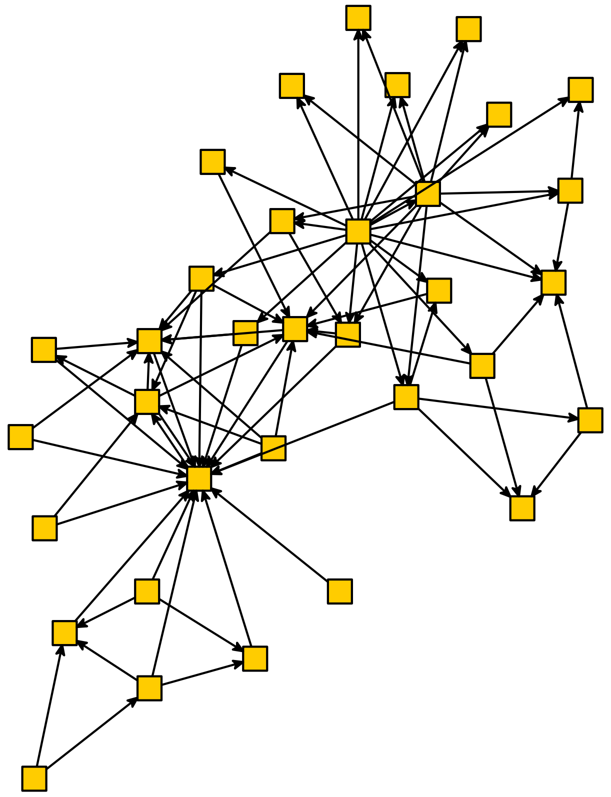
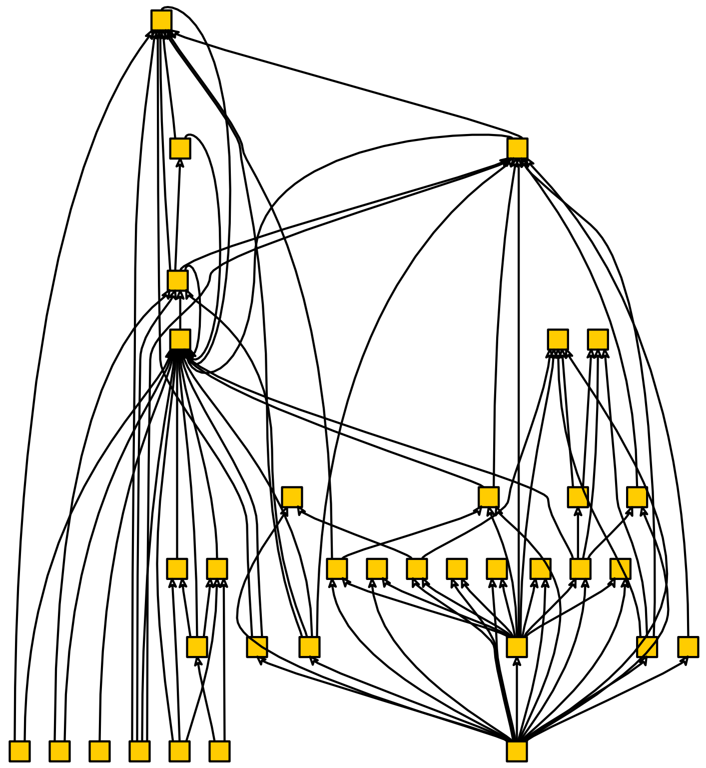
**Given:** directed graph $D = (V, A)$

**Find:** drawing of $D$ that emphasized the hierarchy





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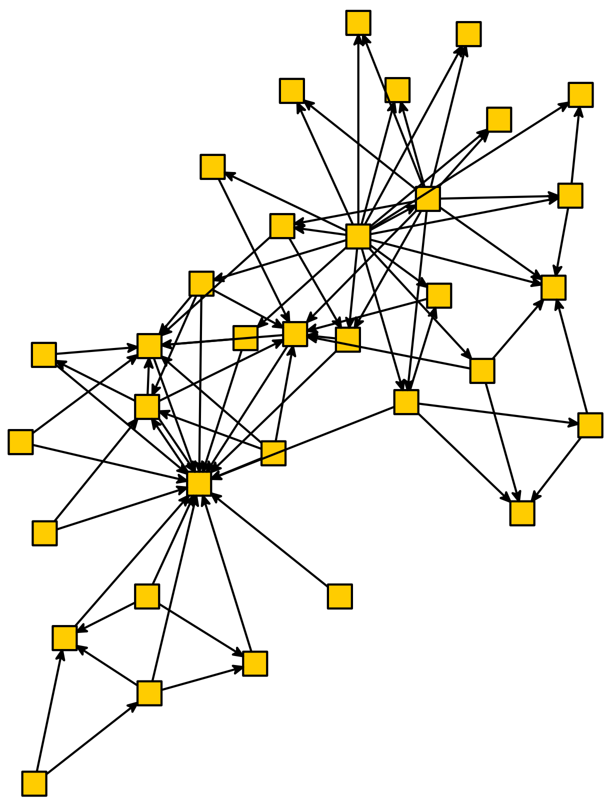

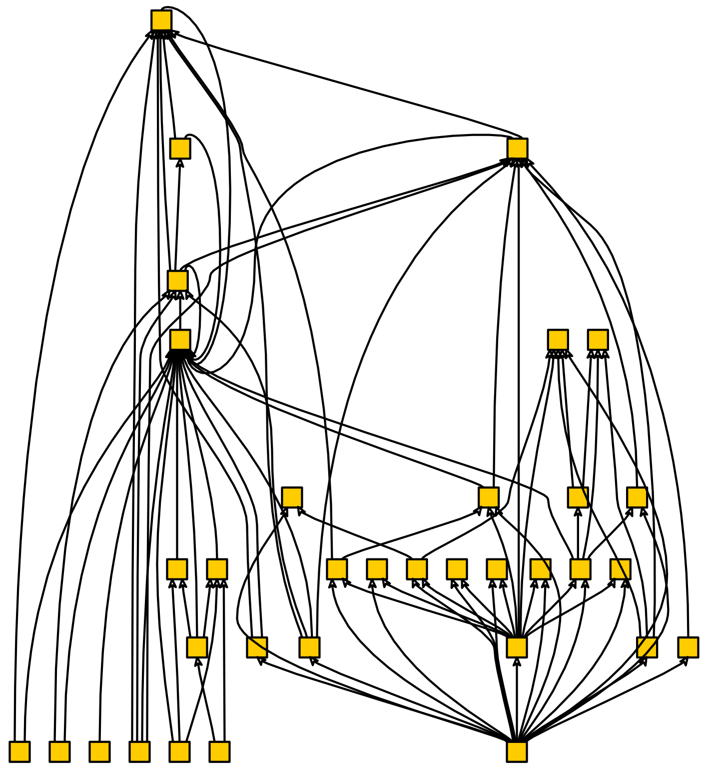

- many edges pointing to the same direction
- nodes lie on (few) horizontal lines

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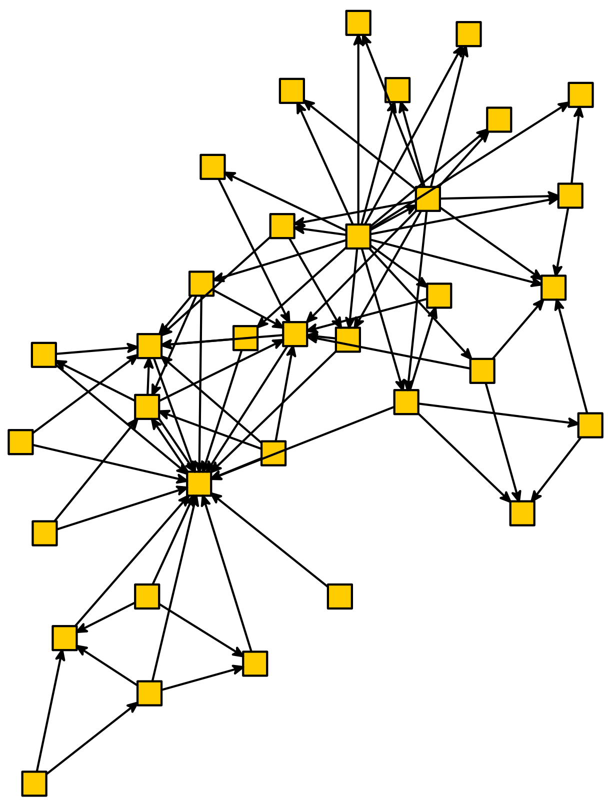
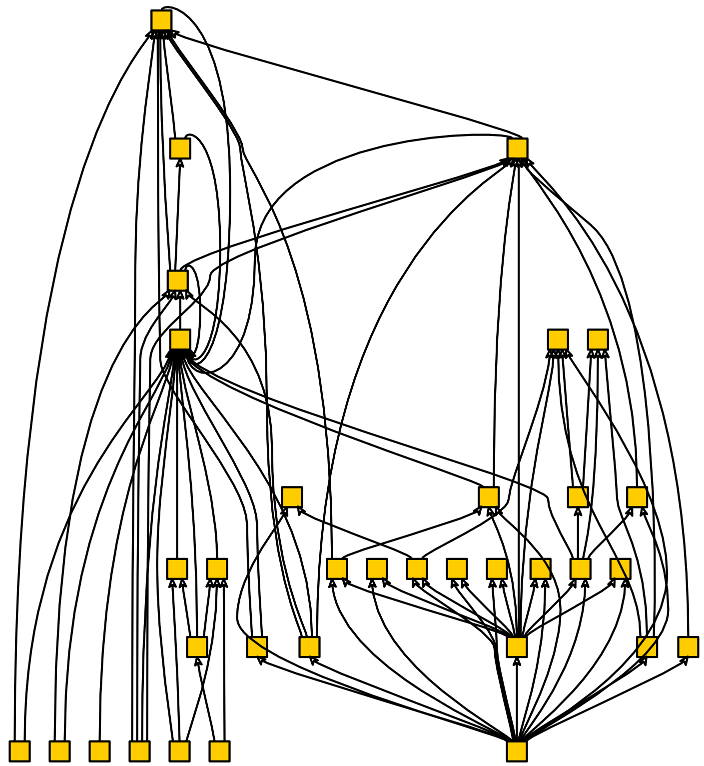
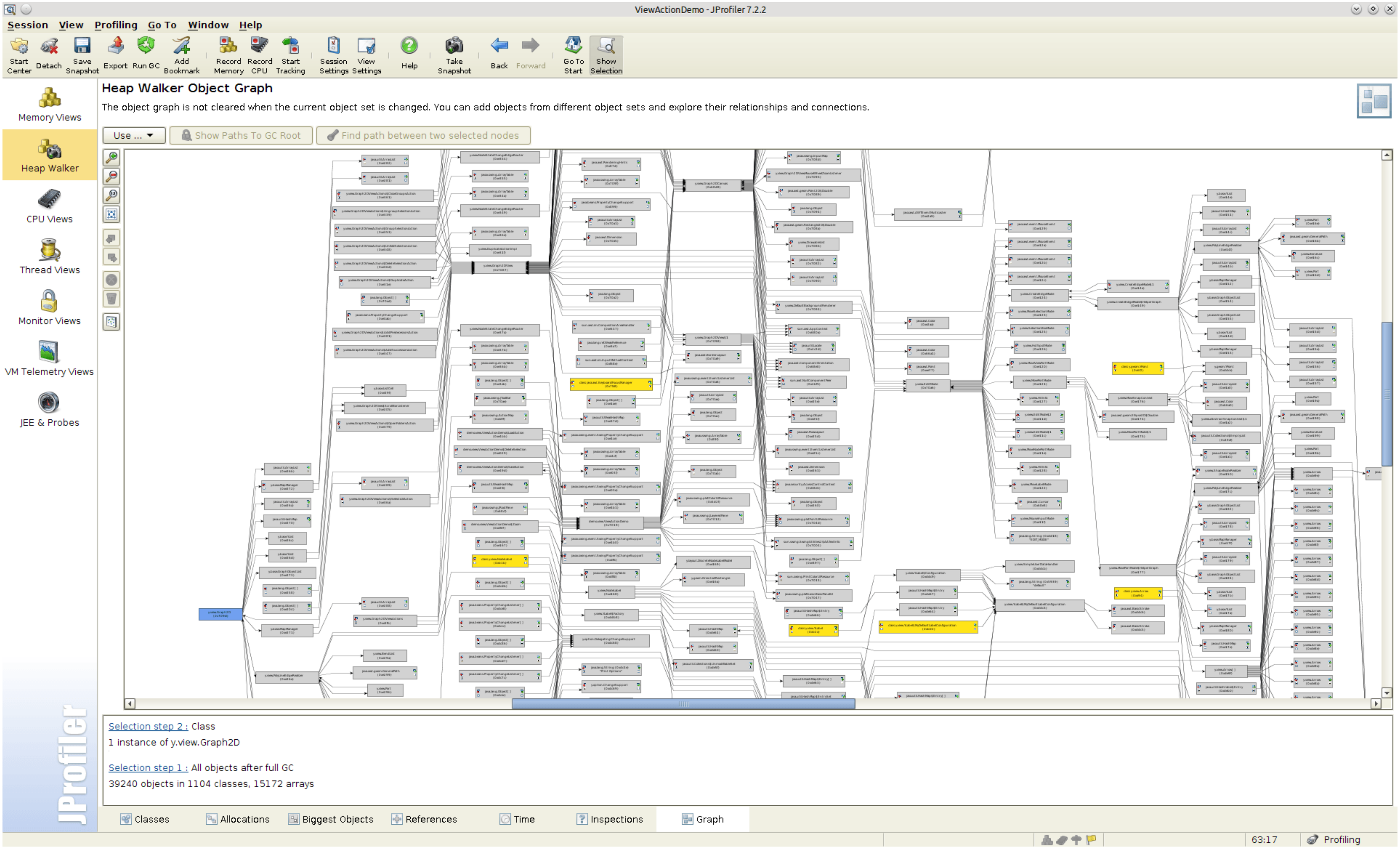




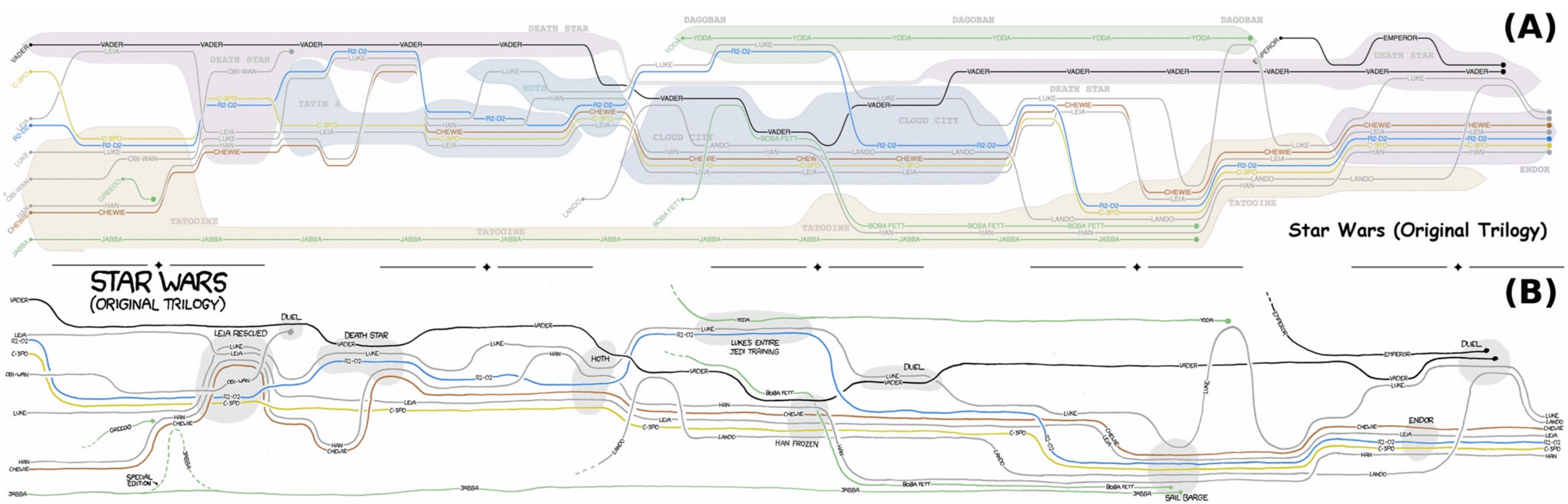
- edges as straight as possible and short
- few edge crossings
- nodes distributed evenly

# Application: Java Profiler




yEd Gallery: Java profiler JProfiler using yFiles

# Application: Storylines




Source: "Design Considerations for Optimizing Storyline Visualizations" Tanahashi et al.

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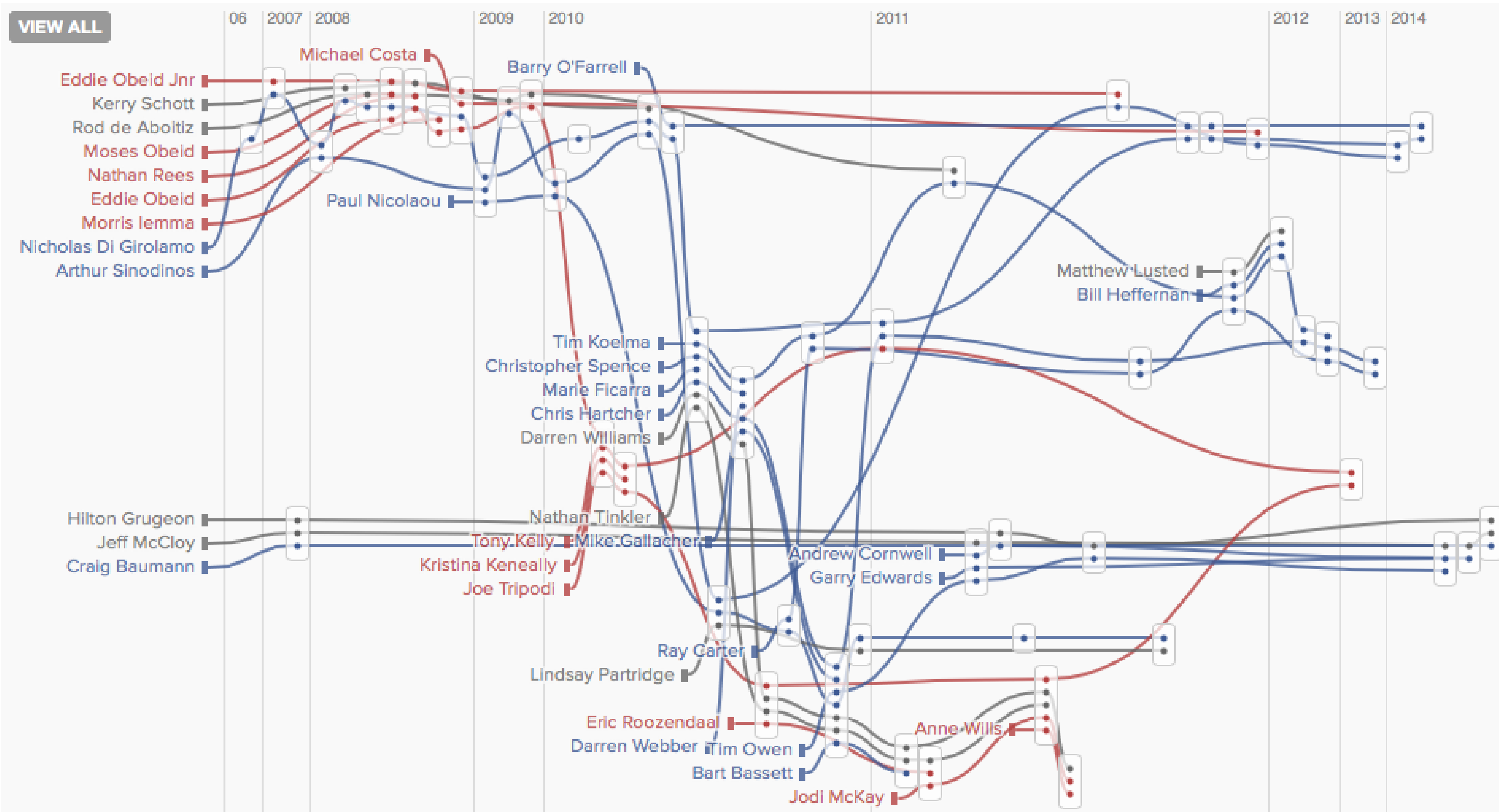
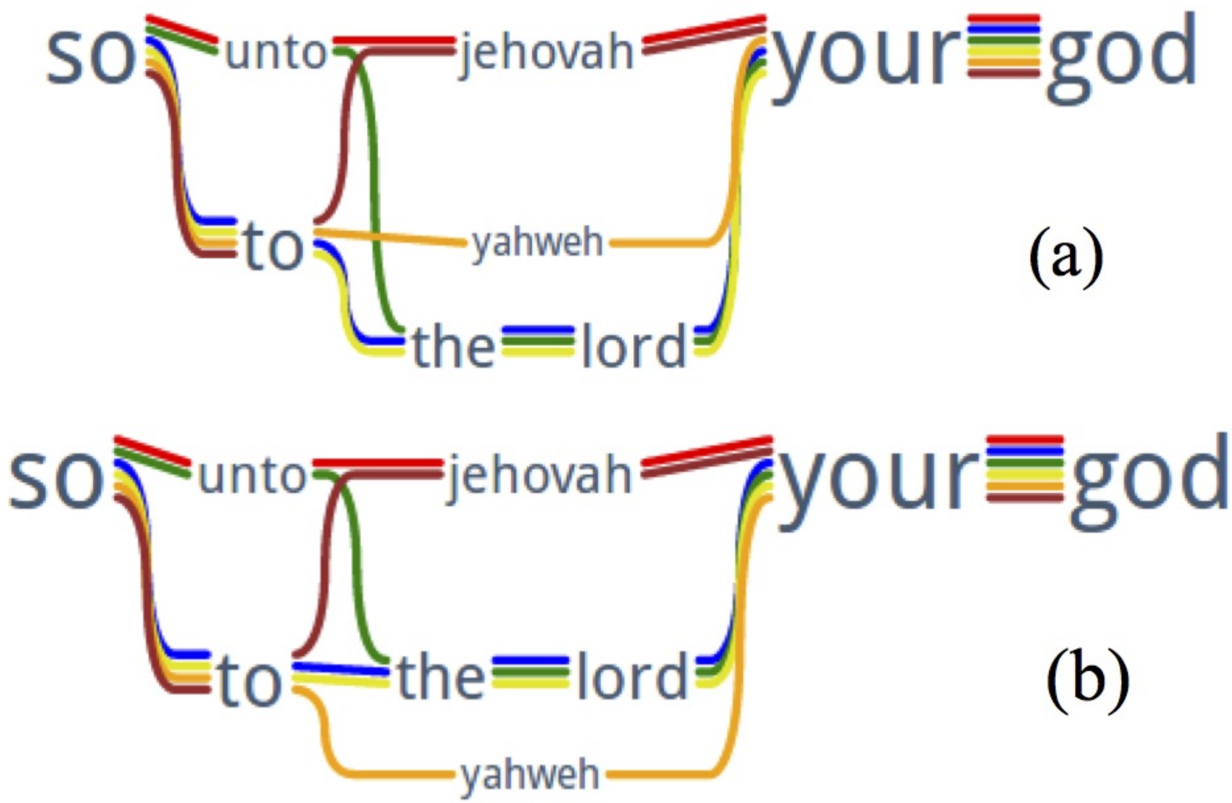



Source: ABC news, Australia

# Application: Text-Variant graphs




Source: Improving the Layout for Text Variant Graphs Jänicke et al.

# Application: Mythological Creatures and Gods

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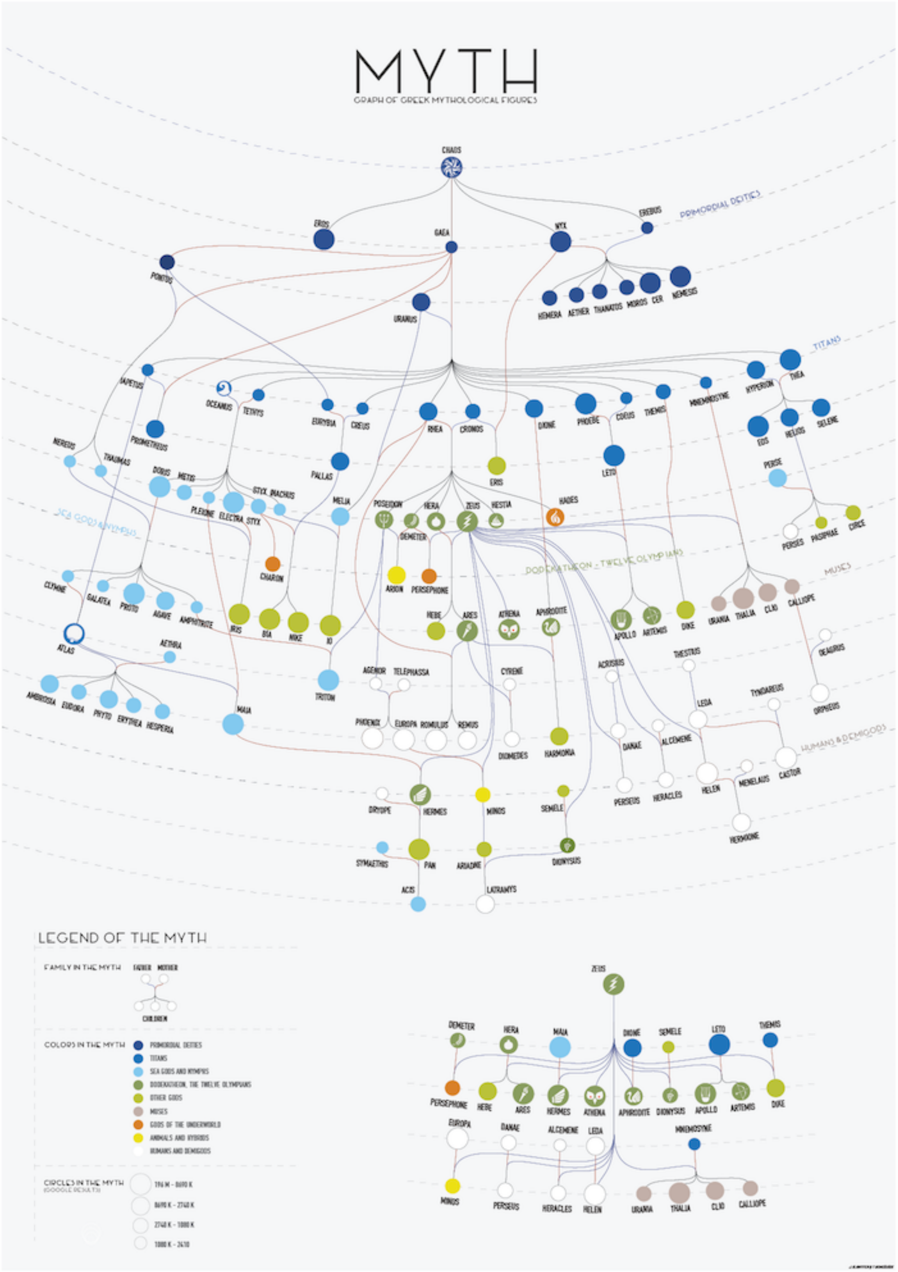
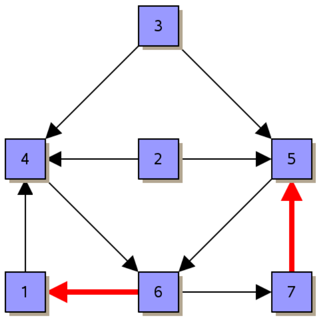



Source: Visualization that won the Graph Drawing contest 2016. Klawitter&Mchedlidze

# Sugiyama Framework (Sugiyama, Tagawa, Toda 1981)

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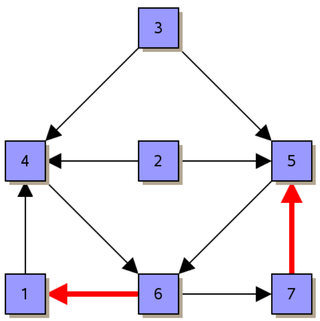


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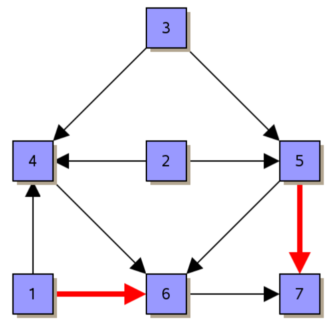


resolve cycles

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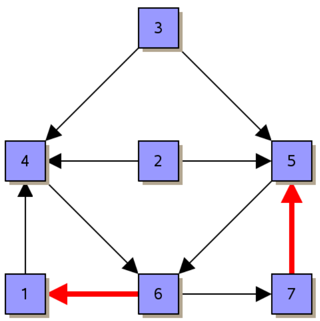


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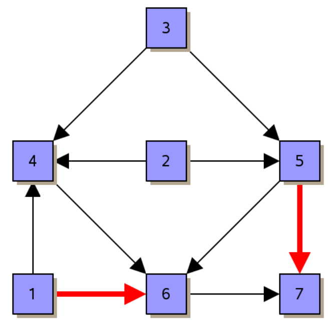


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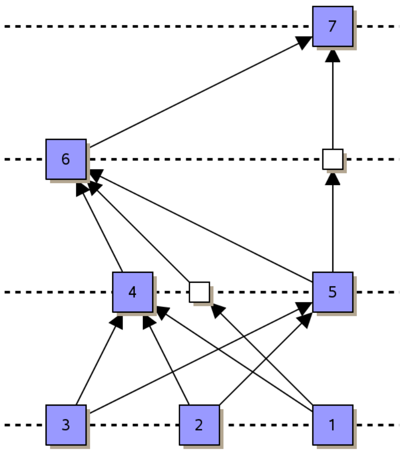


layer assignement

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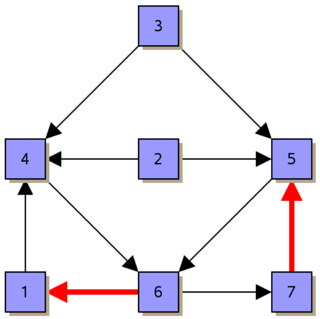


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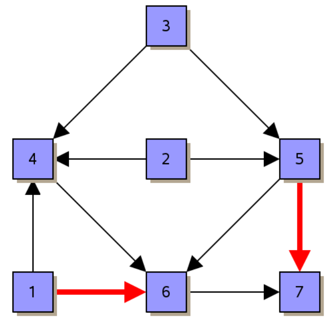


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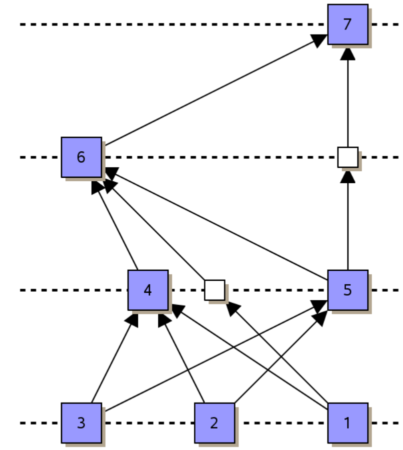


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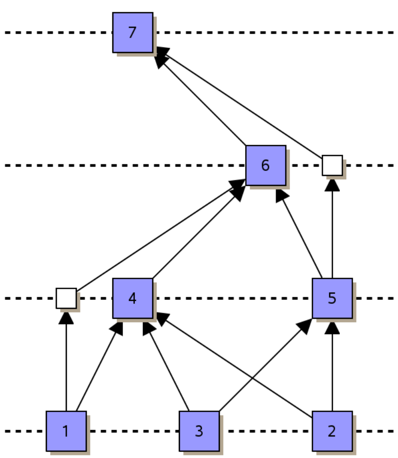


crossing minimization

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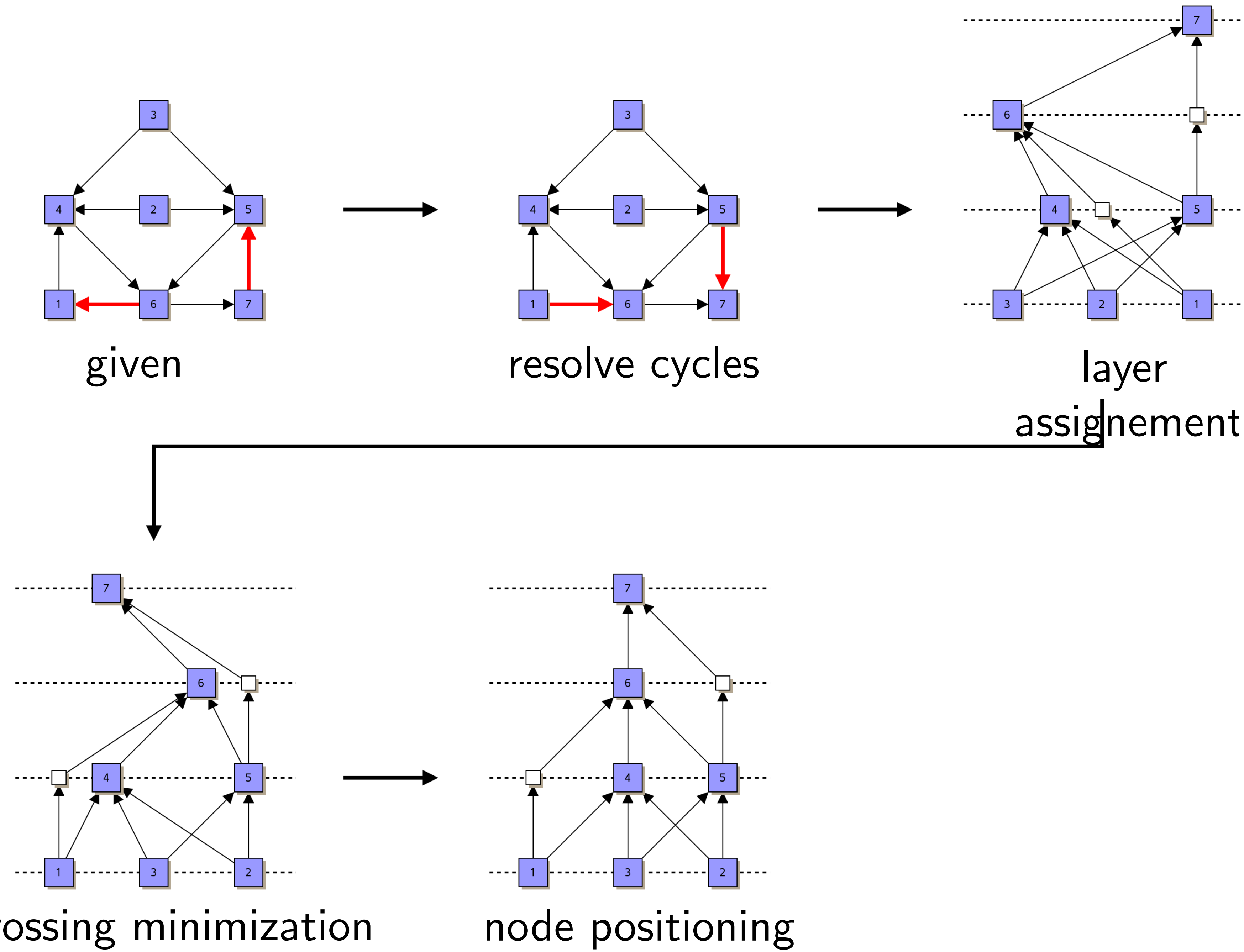

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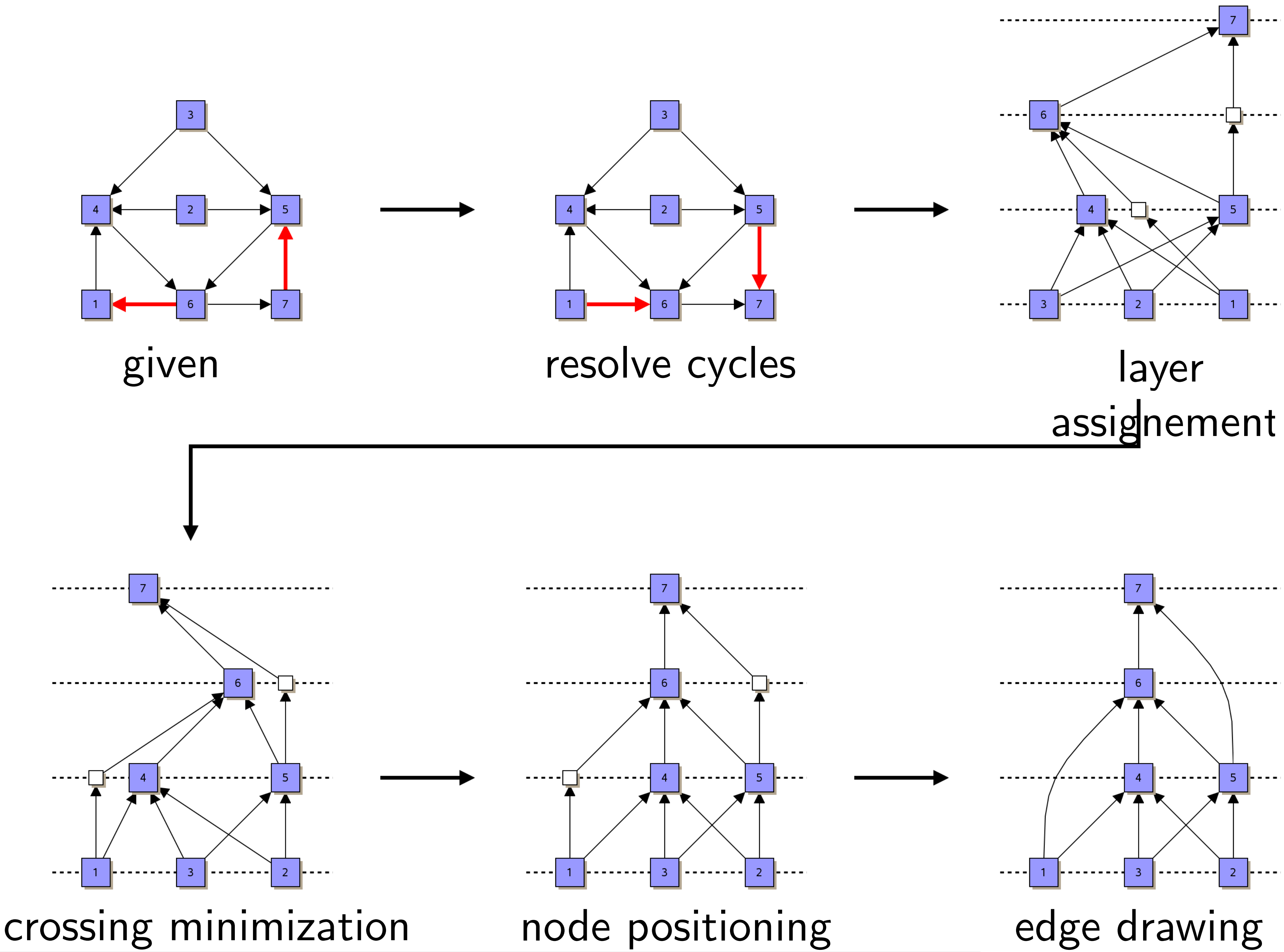

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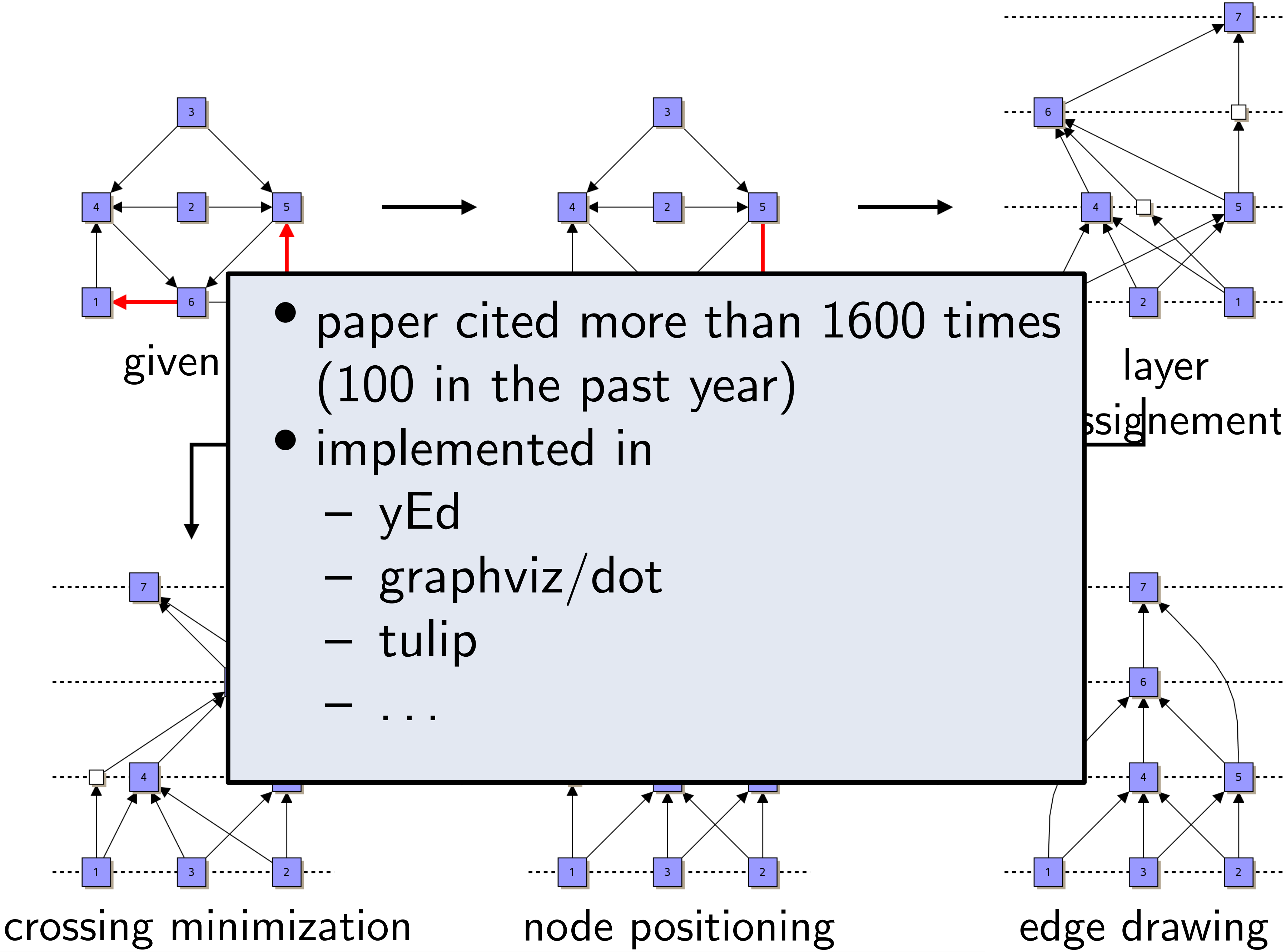


- paper cited more than 1600 times (100 in the past year)
- implemented in
  - yEd
  - graphviz/dot
  - tulip
  - ...

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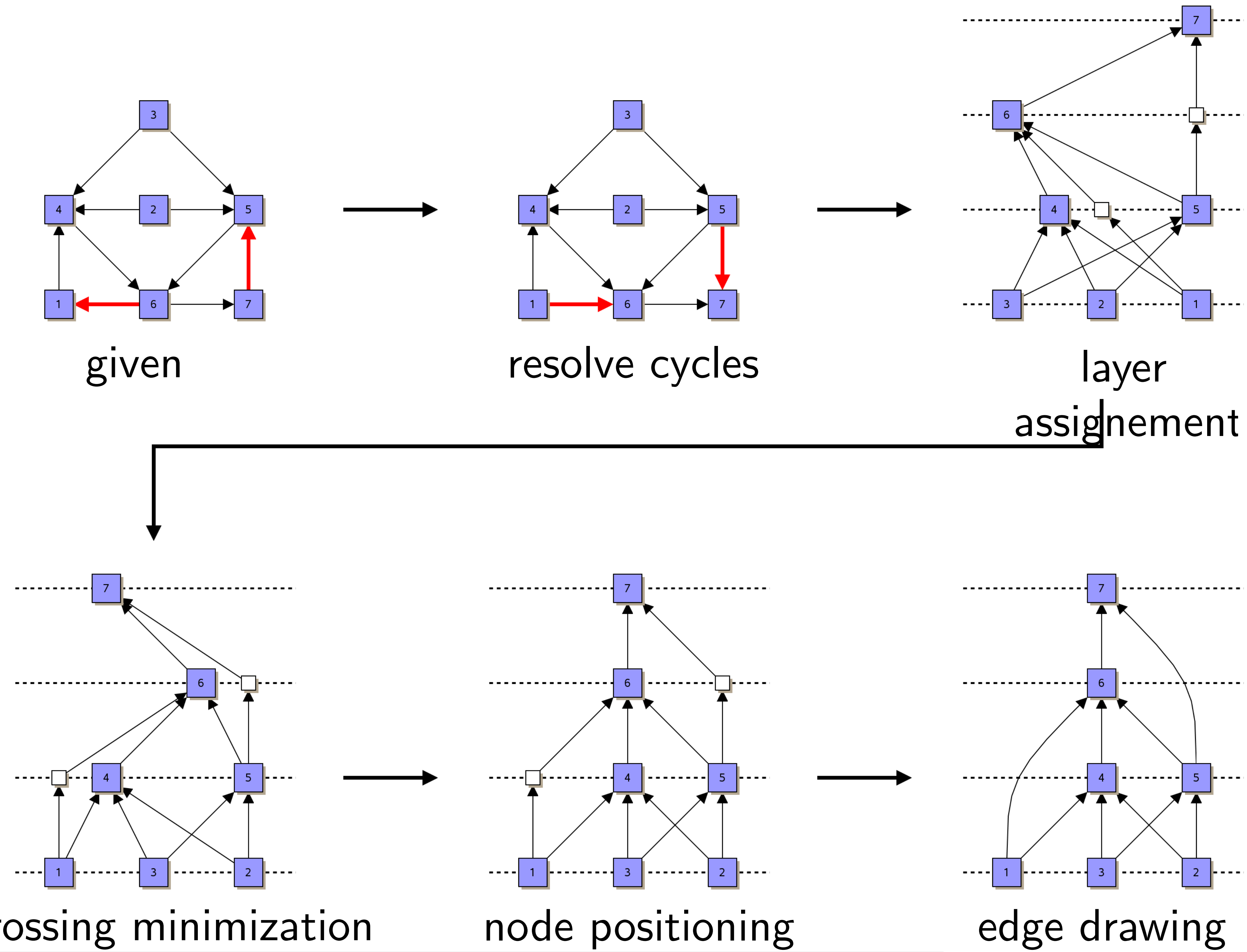

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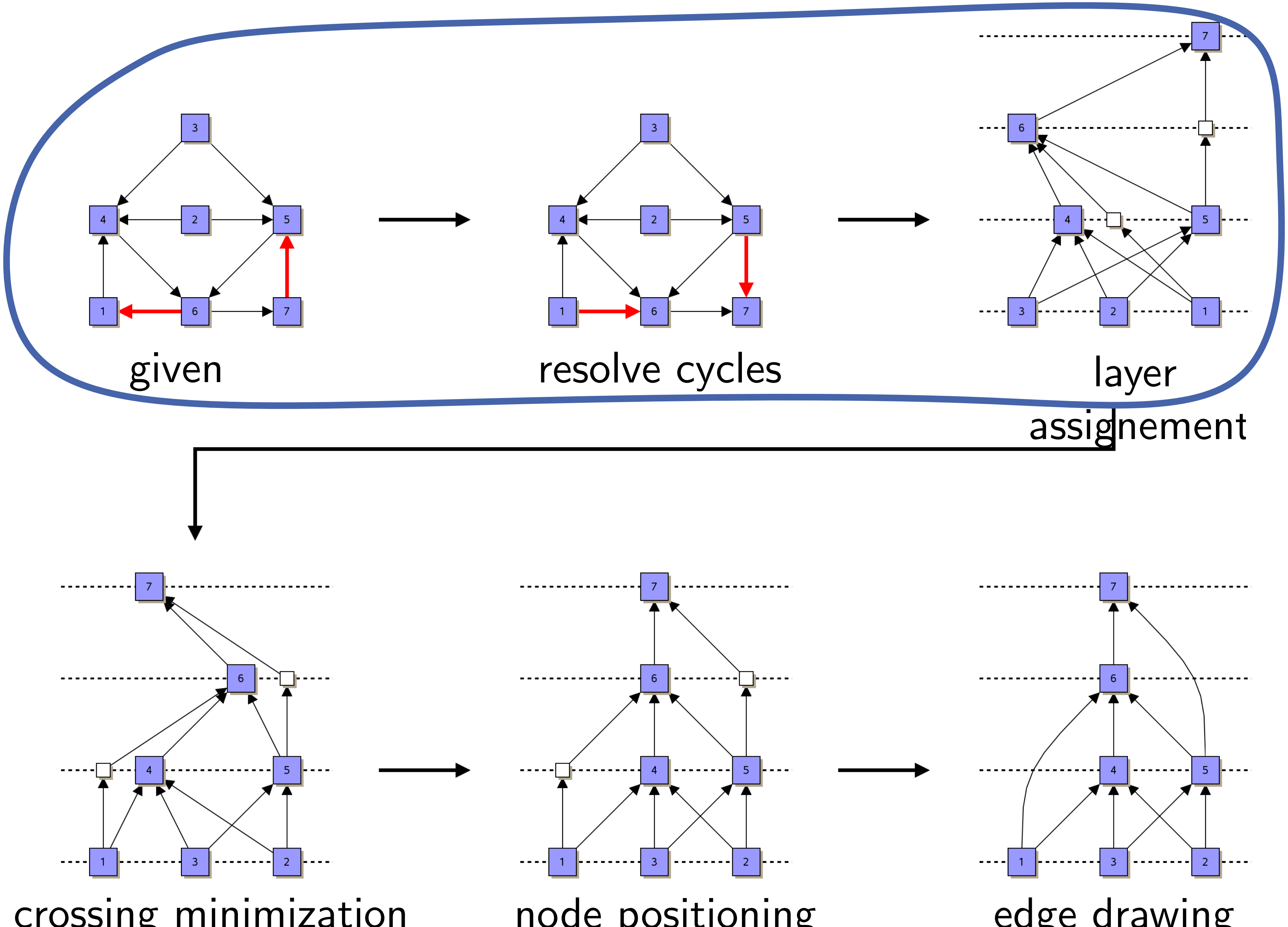
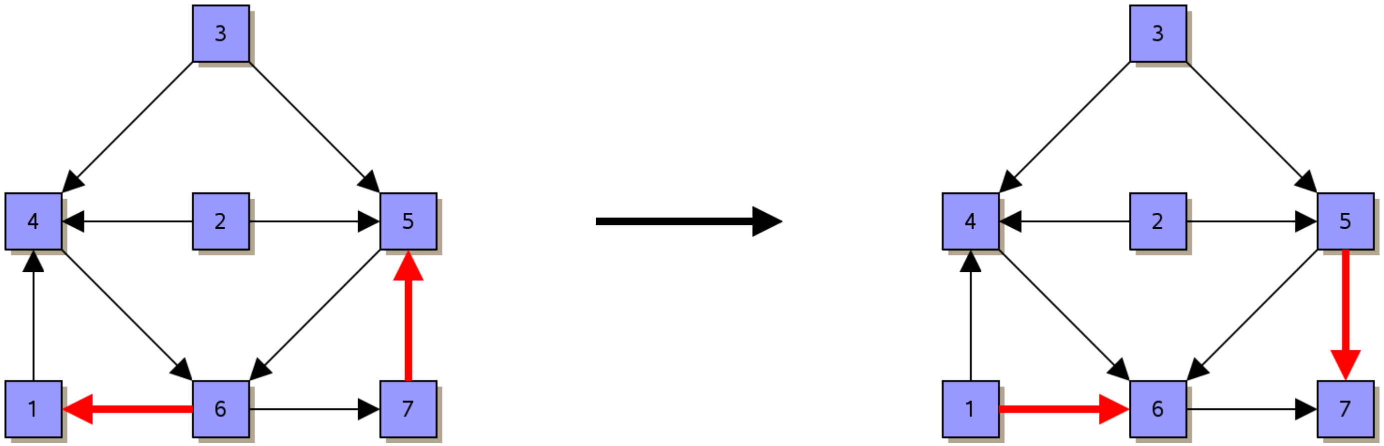


# Step 1: Resolve Cycles



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**All three problems are NP-hard!**

# Heuristic 1 (Berger, Shor 1990)

$A' := \emptyset$;

**foreach** $v \in V$ **do**

  **if** $|N^{\rightarrow}(v)| \geq |N^{\leftarrow}(v)|$ **then**

    $A' := A' \cup N^{\rightarrow}(v)$;

  **else**

    $A' := A' \cup N^{\leftarrow}(v)$;

  remove $v$ and $N(v)$ from $D$.

**return** $(V, A')$

$$N^{\rightarrow}(v) := \{(v,u) : (v,u) \in A\}$$
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**Work with your neighbour(s) and then share**

**5 min**

Why $D'$ does not contain cycles?
What one can say about $|A'|$ in terms of $|A|$?

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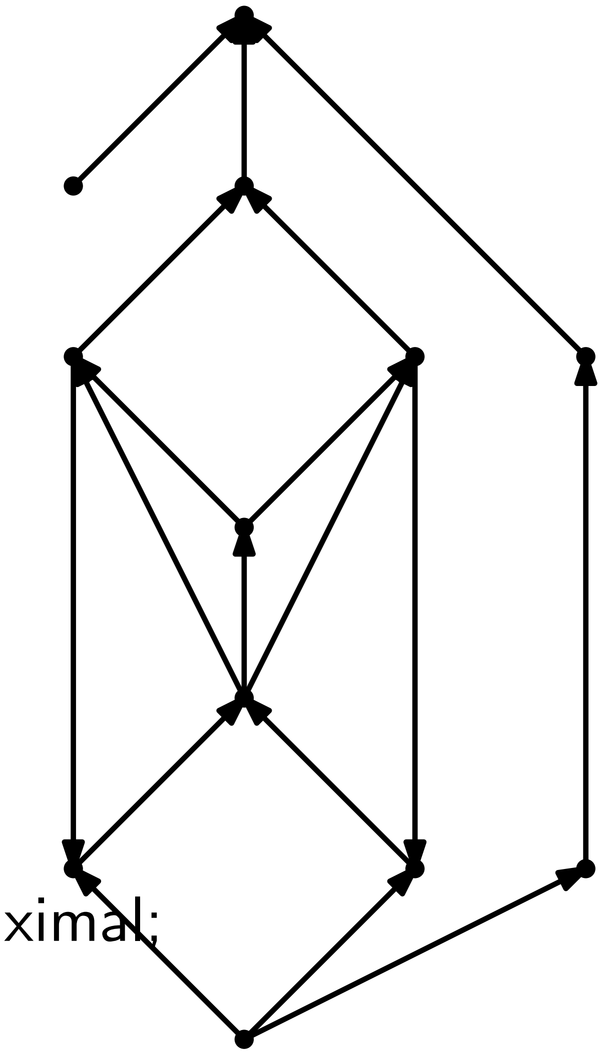

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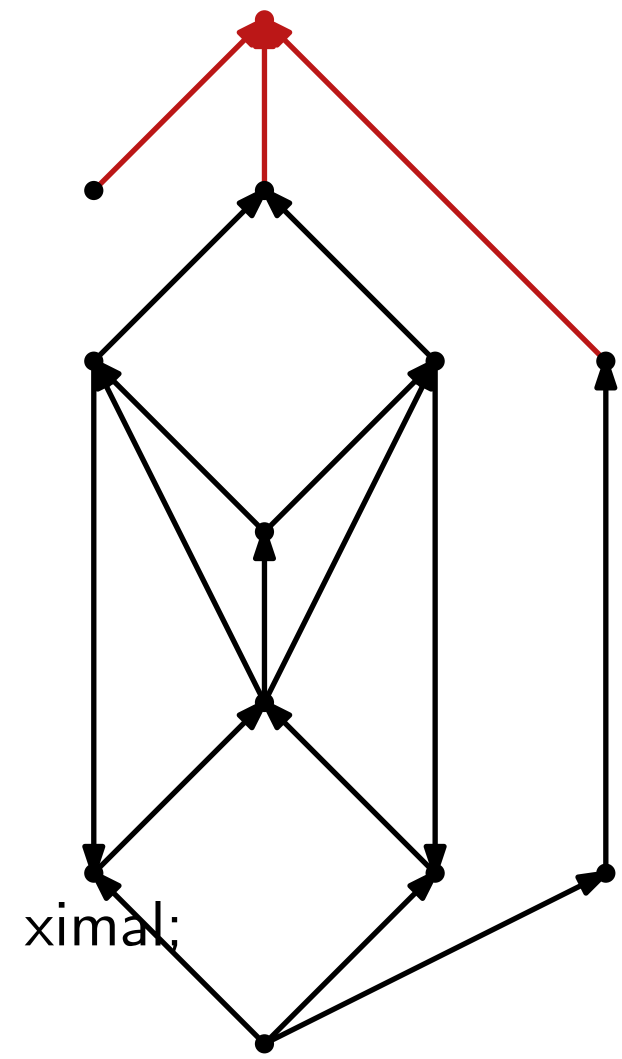

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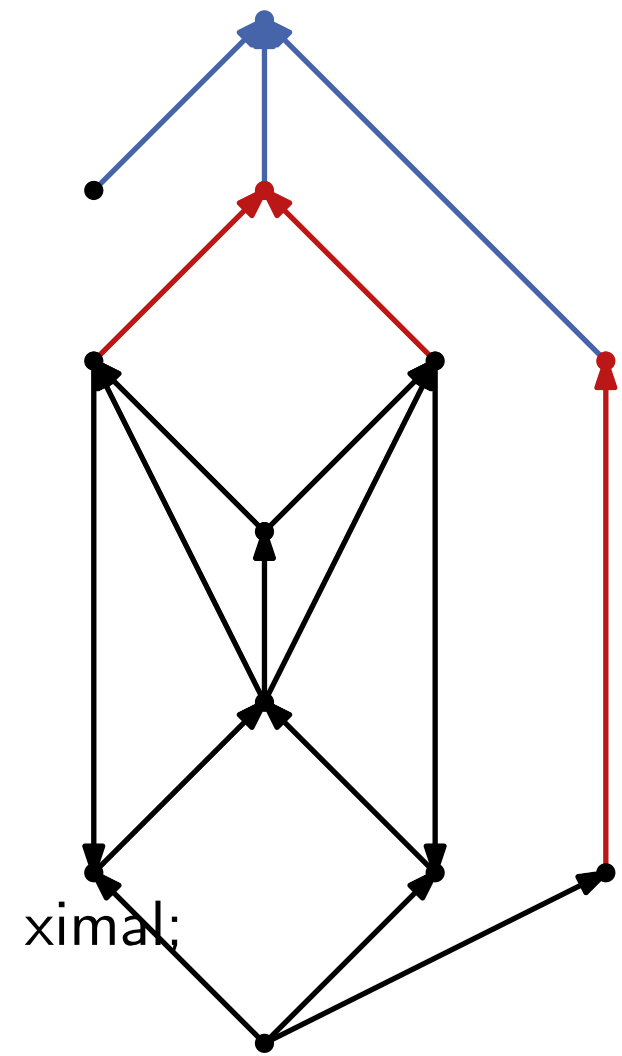

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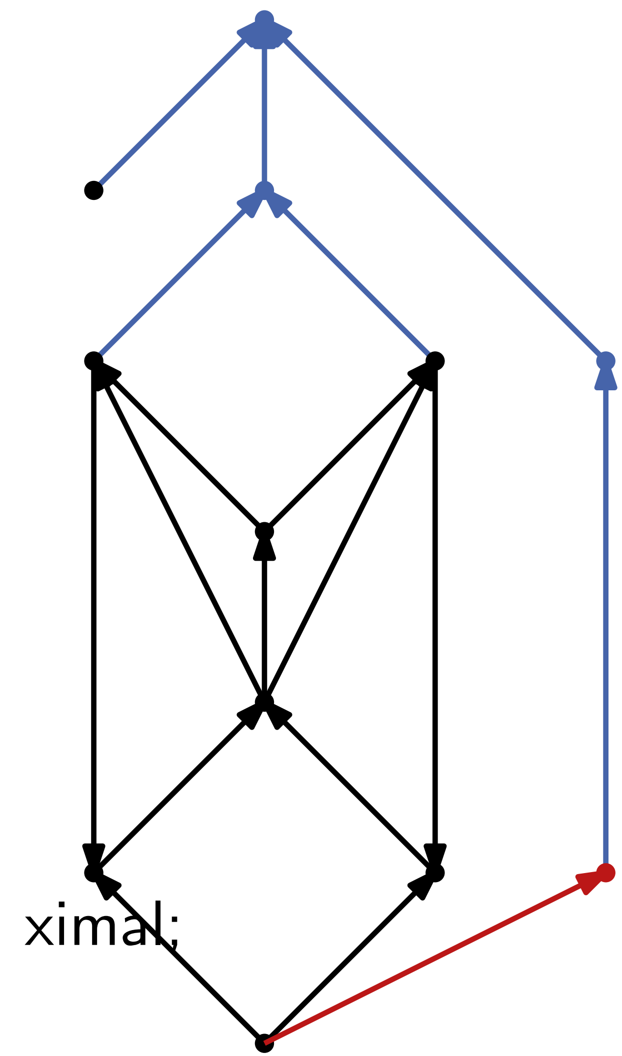

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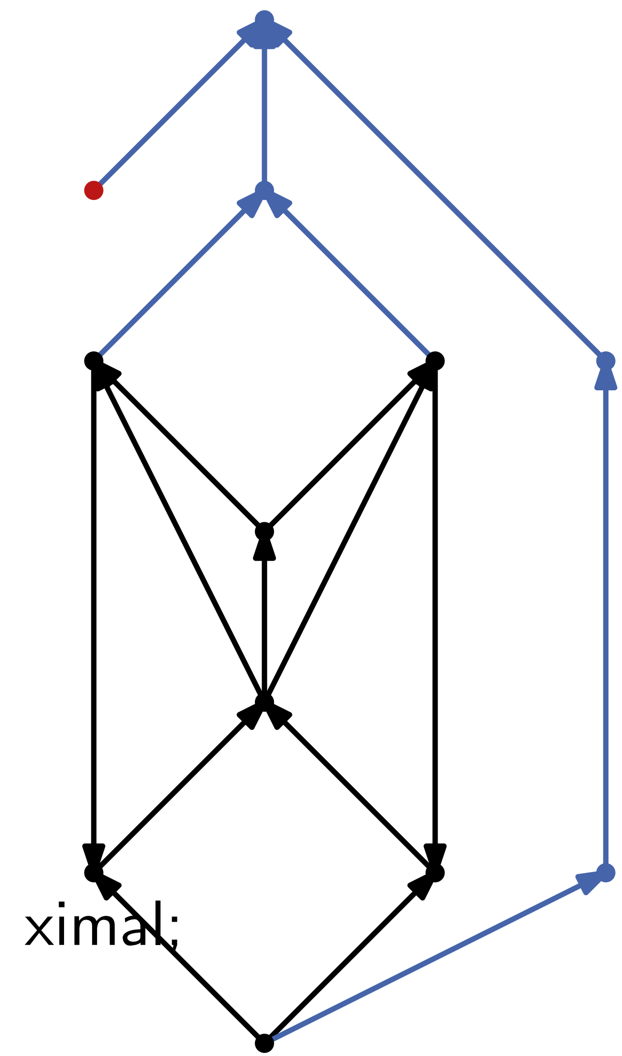

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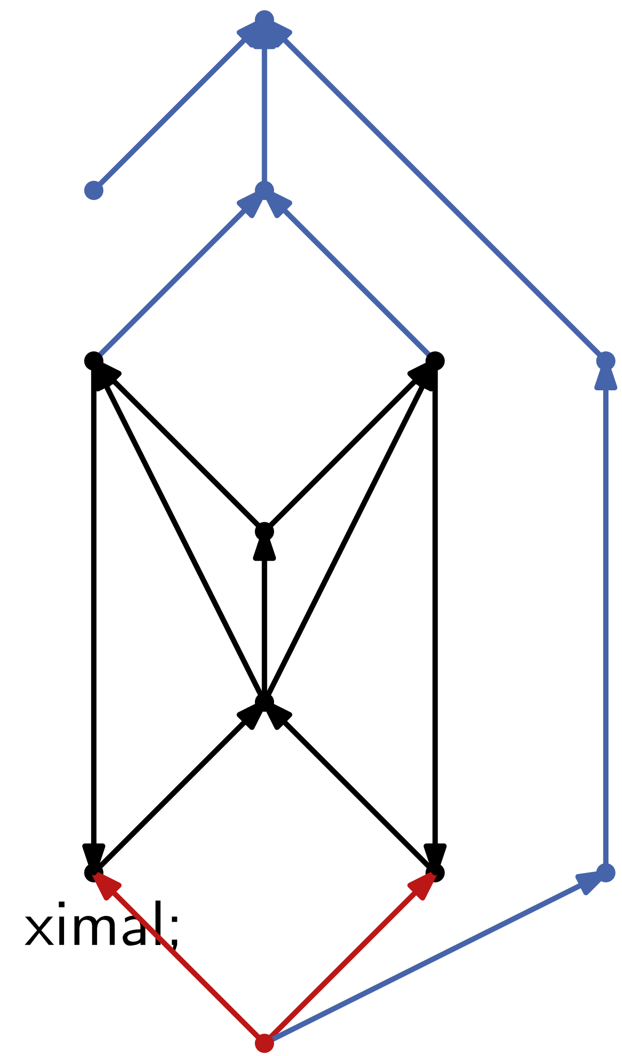

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Karlsruhe Institute of Technology

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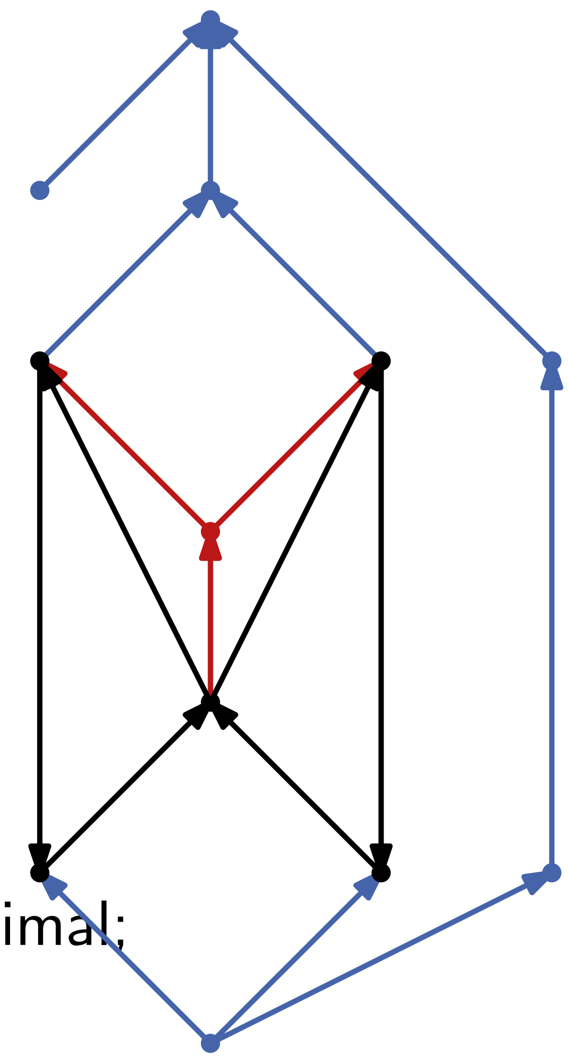

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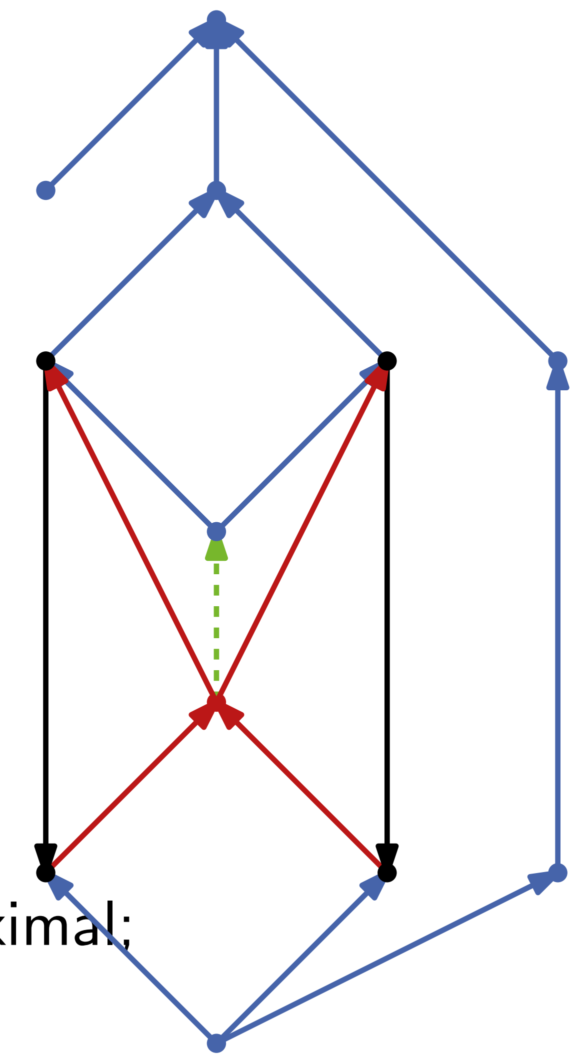

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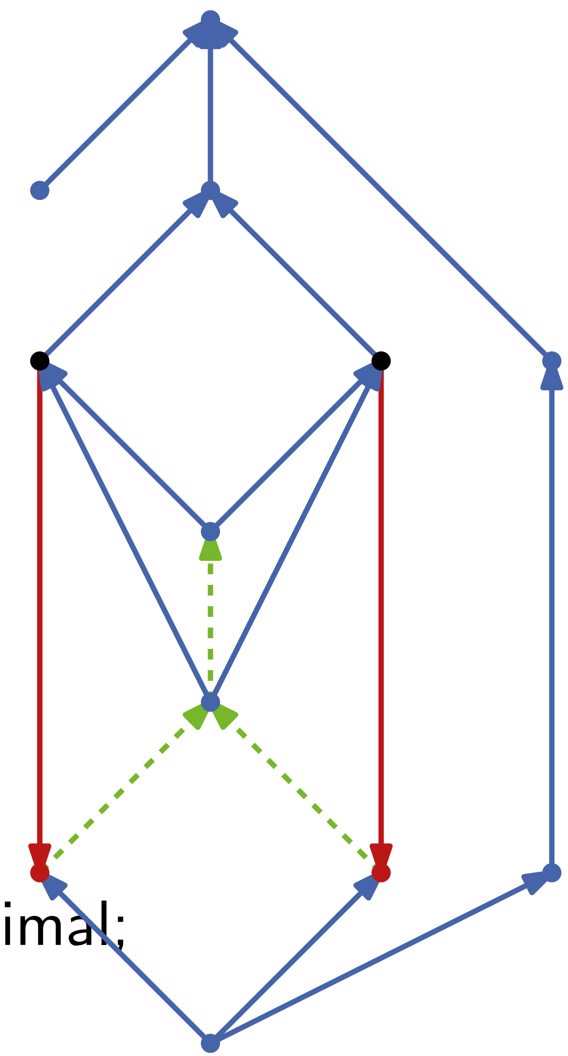

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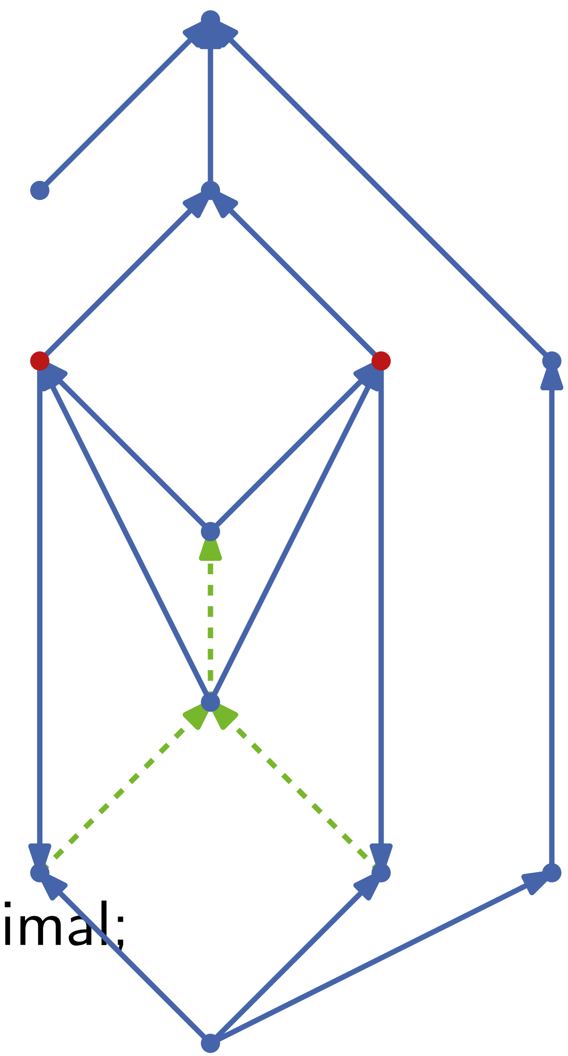

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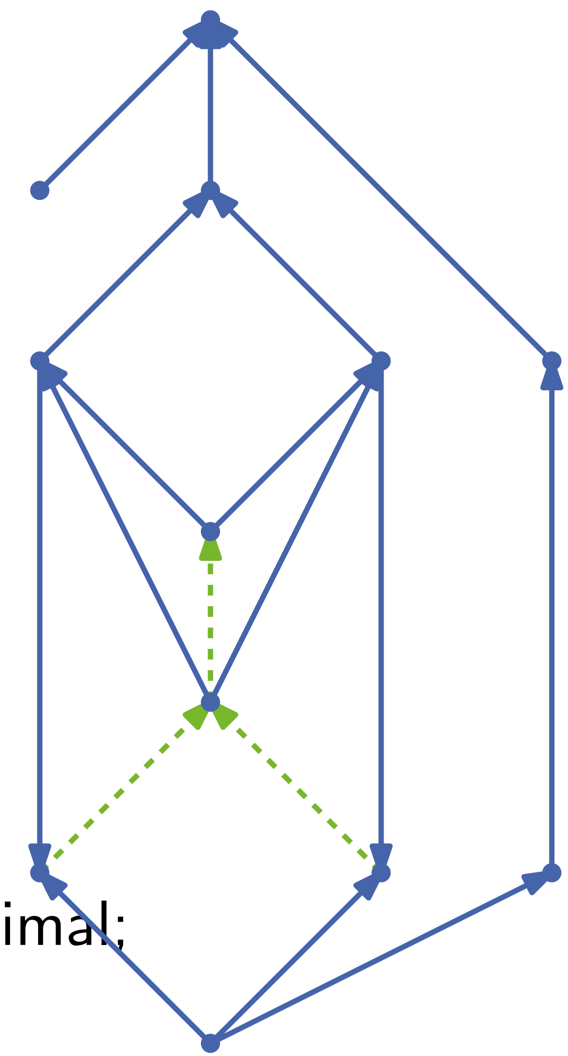


# Heuristic 2 (Eades, Lin, Smyth 1993)

$A' := \emptyset$;

**while** $V \neq \emptyset$ **do**

  **while** in $V$ exists a sink $v$ **do**

    $A' \leftarrow A' \cup N^{\leftarrow}(v)$

    remove $v$ and $N^{\leftarrow}(v)$: $\{V, n, m\}_{\text{sink}}$

  Remove all isolated node from $V$: $\{V, n, m\}_{\text{iso}}$

  **while** in $V$ exists a source $v$ **do**

    $A' \leftarrow A' \cup N^{\rightarrow}(v)$

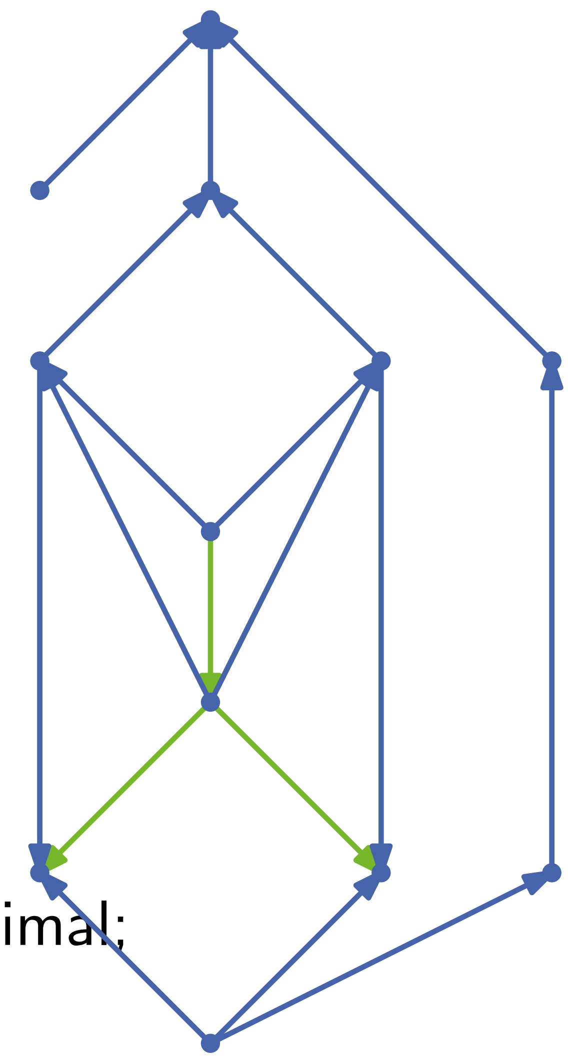
    remove $v$ and $N^{\rightarrow}(v)$: $\{V, n, m\}_{\text{source}}$

  **if** $V \neq \emptyset$ **then**

    let $v \in V$ such that $|N^{\rightarrow}(v)| - |N^{\leftarrow}(v)|$ maximal;

    $A' \leftarrow A' \cup N^{\rightarrow}(v)$

    remove $v$ and $N(v)$: $\{V, n, m\}_{\{=,<\}}$



# Heuristic 2 – Analysis

**Theorem 1:** Let $D = (V, A)$ be a connected, directed graph without 2-cycles. Heuristic 2 computes a set of edges $A'$ with $|A'| \geq |A|/2 + |V|/6$.
The running time is $O(|A|)$.

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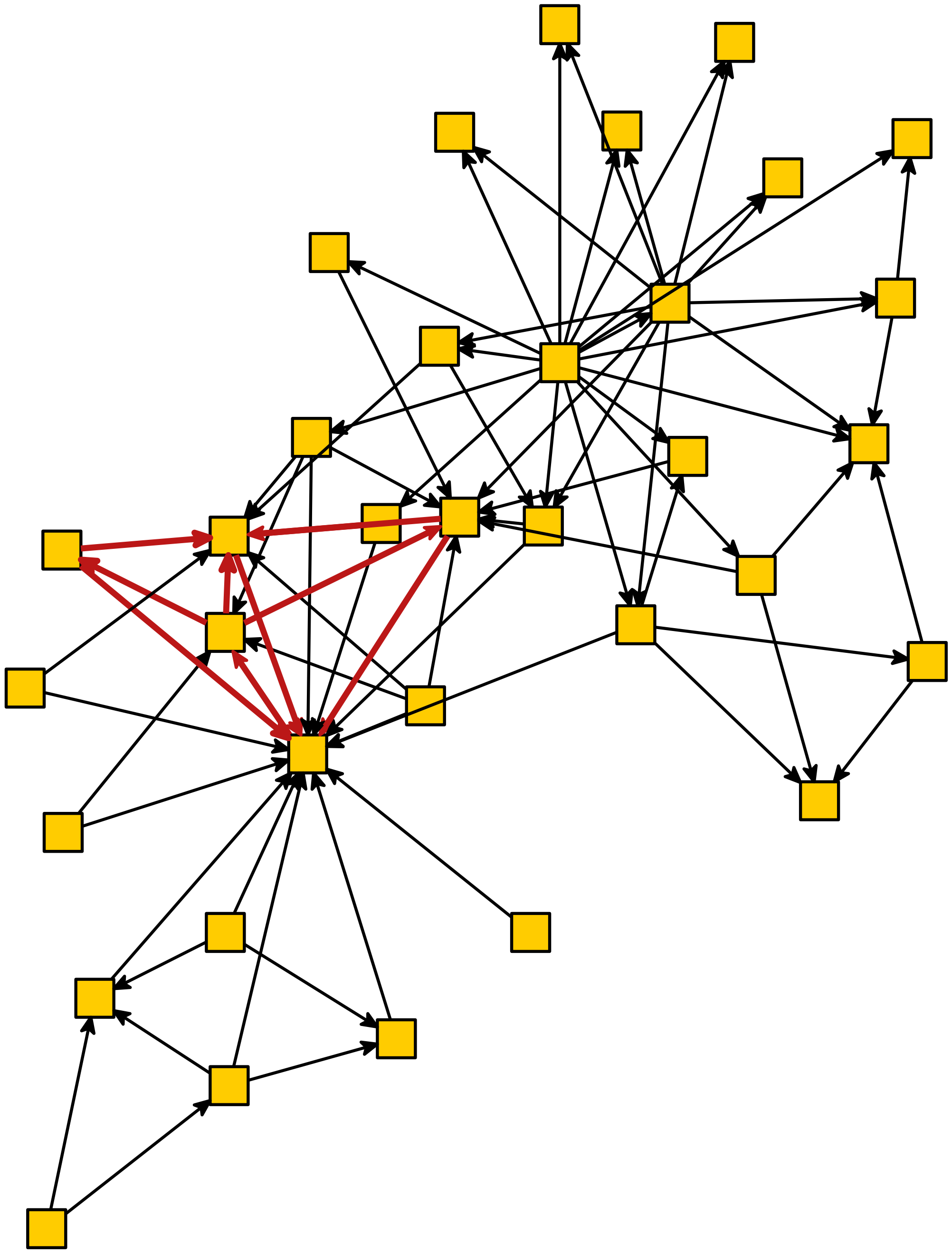
The running time is $O(|A|)$.

**Exact Solution:**

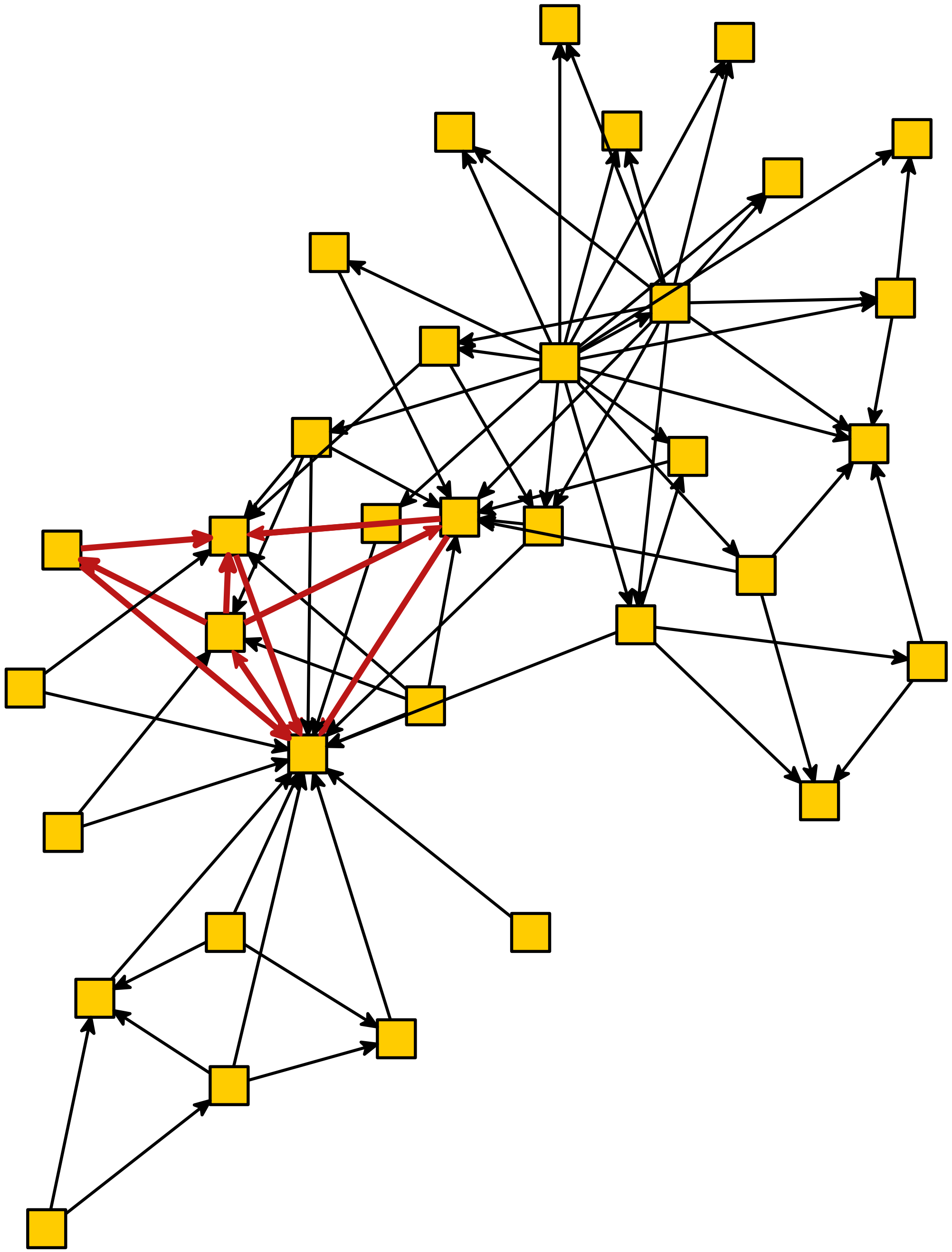
- integer linear programming, using branch-and-cut technique

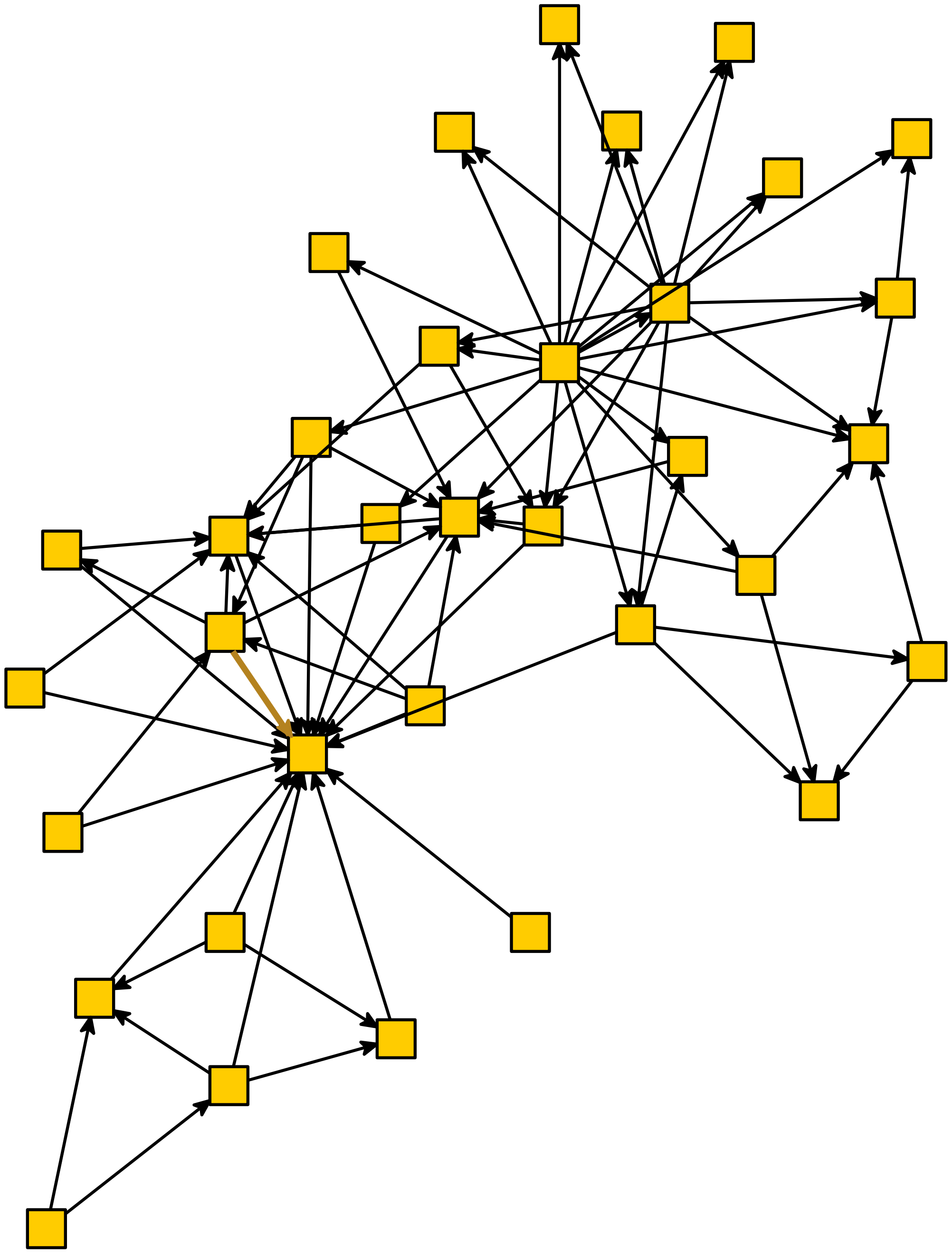
  (Grötschel et al. 1985)

# Example

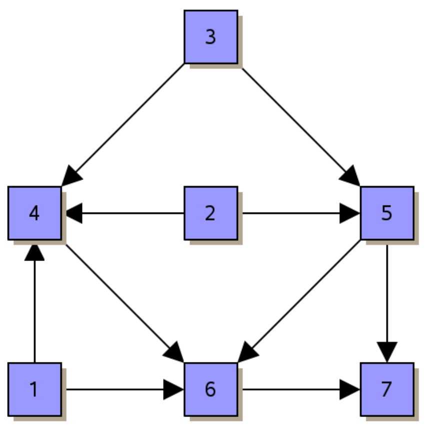


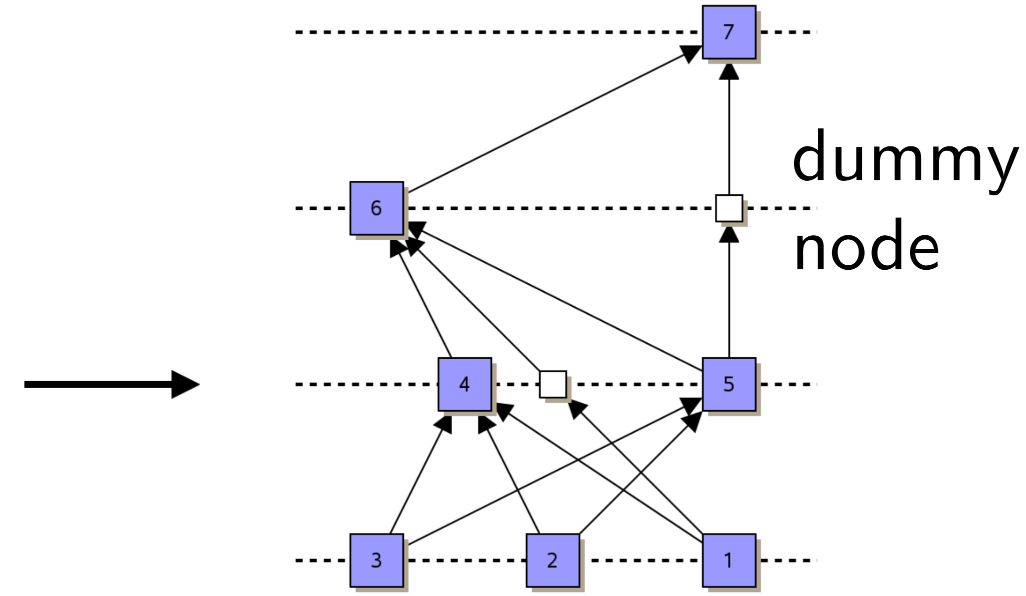
# Example





# Step 2: Layer Assignement



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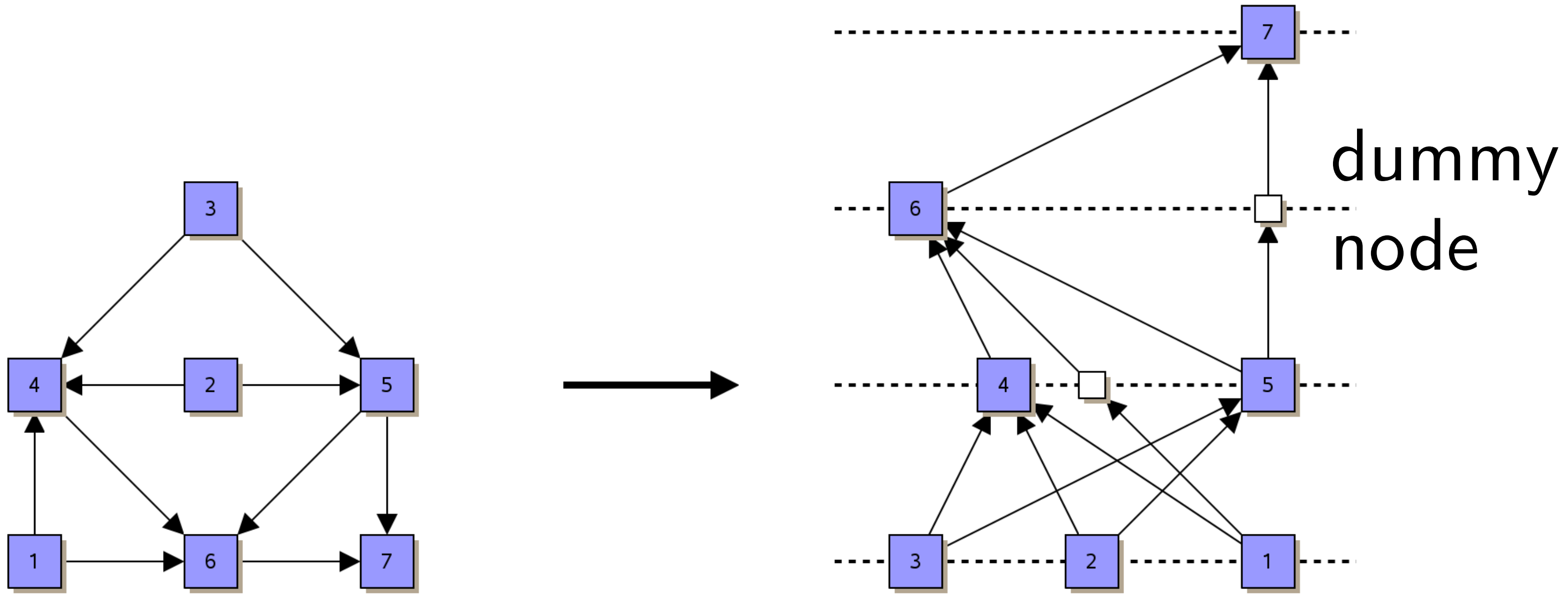


**Think for a minute and then share**

What could we optimize when doing the layer assignment?

**1 min**

# Step 2: Layer Assignement

**Given.:** directed acyclic graph (DAG) $D = (V, A)$

**Find:** Partition the vertex set $V$ into disjoint subsets (**layers**) $L_1, \dots, L_h$ s.t. $(u, v) \in A$, $u \in L_i$, $v \in L_j \Rightarrow i < j$

**Def:** $y$-Coordinate $y(u) = i \Leftrightarrow u \in L_i$

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**Criteria that we discuss**

- minimize the number of layers $h$ (= height of the layouts)
- minimize the total length of edges ($\approx$ number of dummy nodes)
- minimize width, e.g. $\max\{|L_i| \mid 1 \leq i \leq h\}$

# Height Optimization

**Idea:** assign each node $v$ to the layer $L_i$, where $i$ is the length of the longest simple path from a source to $v$

- all incomming neighbours lie below $v$
- the resulting height $h$ is minimized

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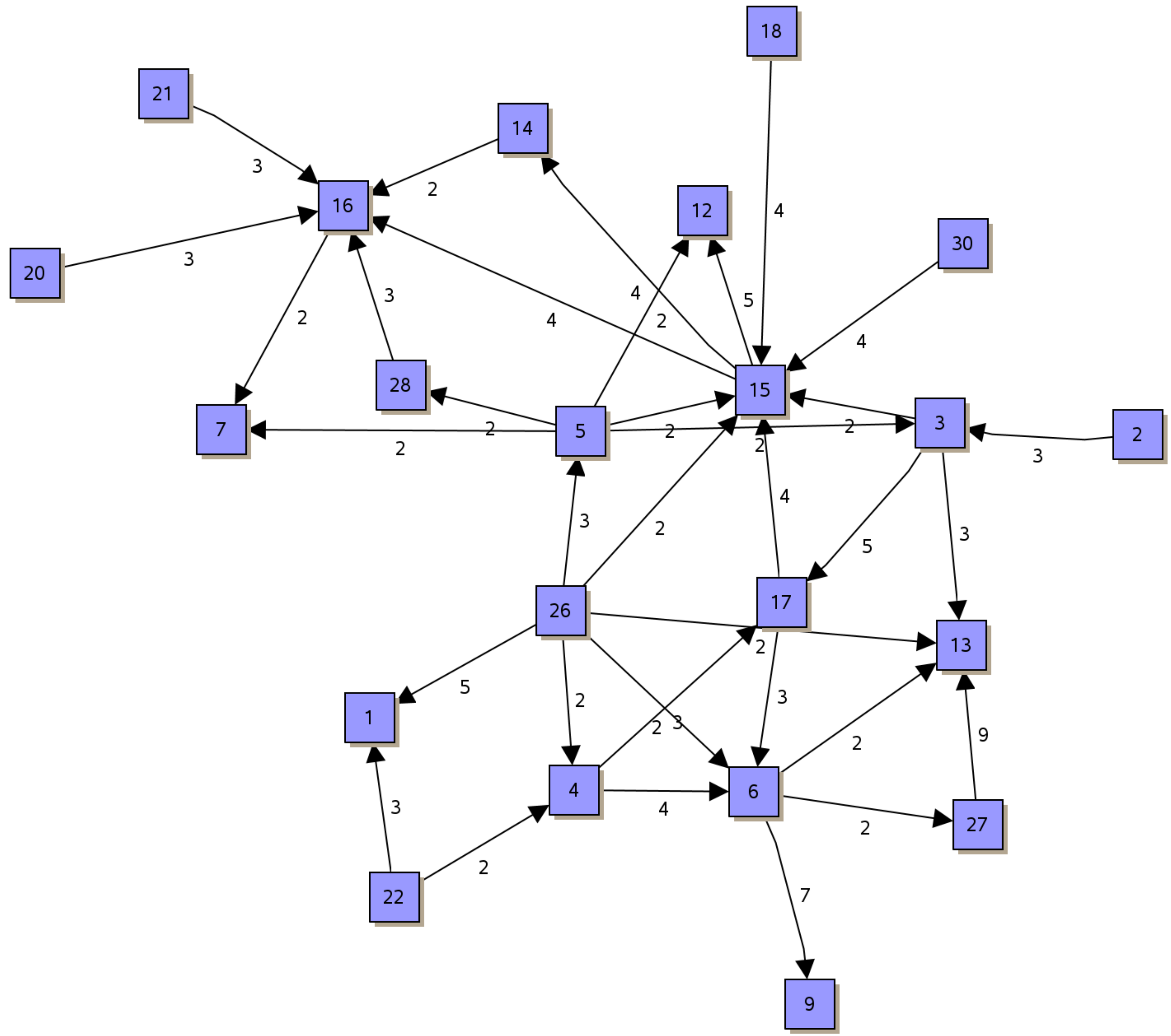
## Algorithm

- $L_1 \leftarrow$ the set of sources in $D$
- set $y(u) \leftarrow \max_{v \in N^{\leftarrow}(u)}\{y(v)\} + 1$

# Example



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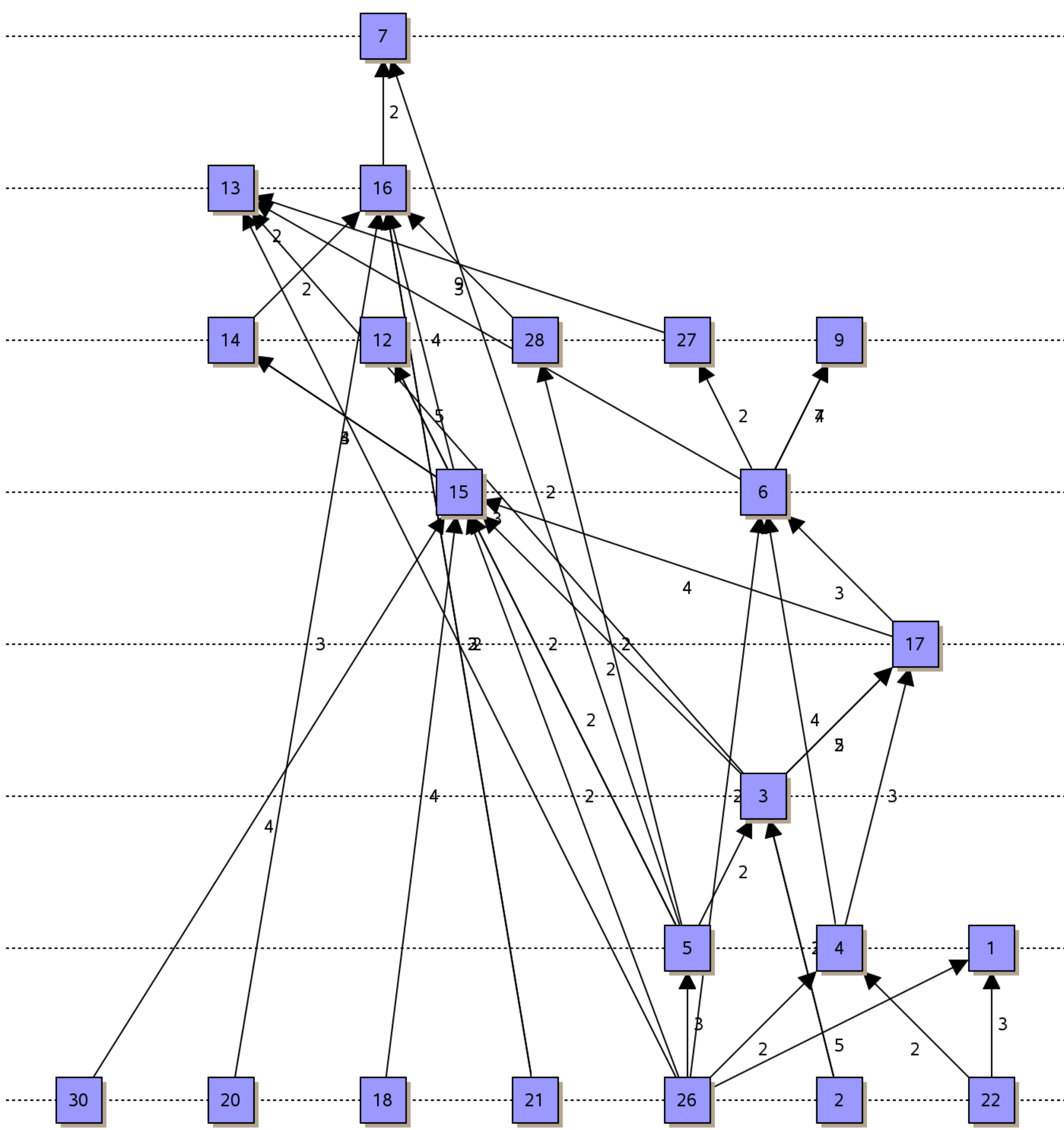
# Example



# Total Edge Length

Can be formulated as an integer linear program:

$$
\begin{array}{rll}
\text{min} & \sum_{(u,v)\in A}(y(v)-y(u)) & \\
\text{subject to} & y(v)-y(u)\geq 1 & \forall (u,v)\in A \\
& y(v)\geq 1 & \forall v\in V \\
& y(v)\in \mathbb{Z} & \forall v\in V
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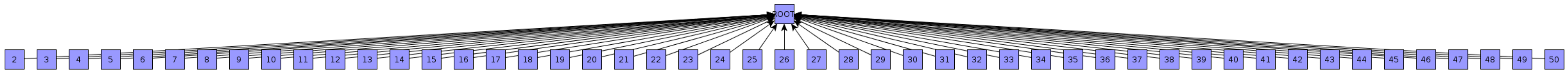
\end{array}
$$

One can show that:

- Constraint-Matrix is **totally unimodular**
- $\Rightarrow$ Solution of the relaxed linear program is integer
- The total edge length can be minimized in a polynomial time

# Width of the Layout



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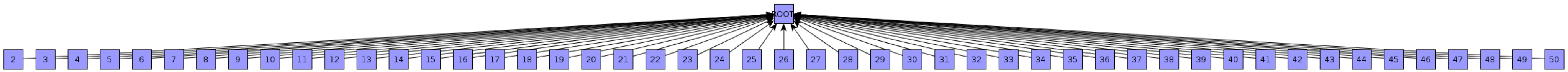


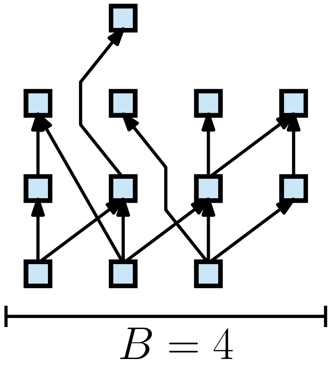

→ bound the width!

# Layer Assignement with Fixed Width

**Fixed-Width Layer Assignment**

**Given:** directed acyclic graph $D = (V, A)$, width $B$

**Find:** layer assignement $\mathcal{L}$ of minumum height with at most $B$ nodes per layer



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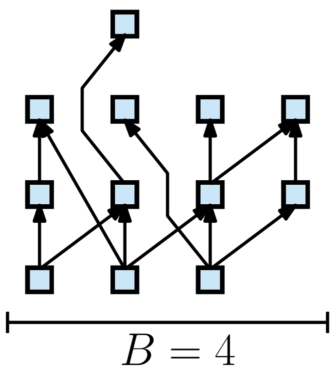


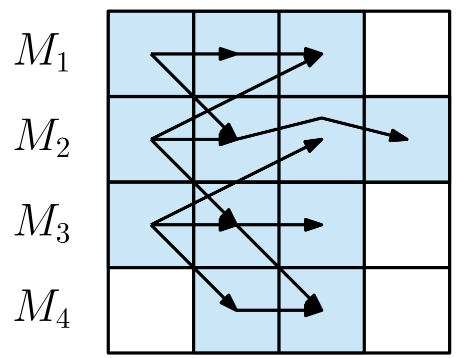


$\rightarrow$ equivalent to the following scheduling problem:

## Minimum Precedence Constrained Scheduling (MPCS)

**Given:** $n$ Jobs $J_1, \dots, J_n$ with identical unit processing time, precedence constraints $J_i < J_k$, and $B$ identical machines

**Find:** Schedule of minimum length, that satisfies all the precendence constraints

# Fixed Width: Complexity

**Theorem 2:** It is NP-hard to decide, whether for $n$ jobs $J_1, \ldots, J_n$ of identical length, given partial ordering constraints, and number of machines $B$, there exists a schedule of height at most $T$, even if $T = 3$.

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**Work with your neighbour(s) and then share**

Why the corollary holds?

**2 min**

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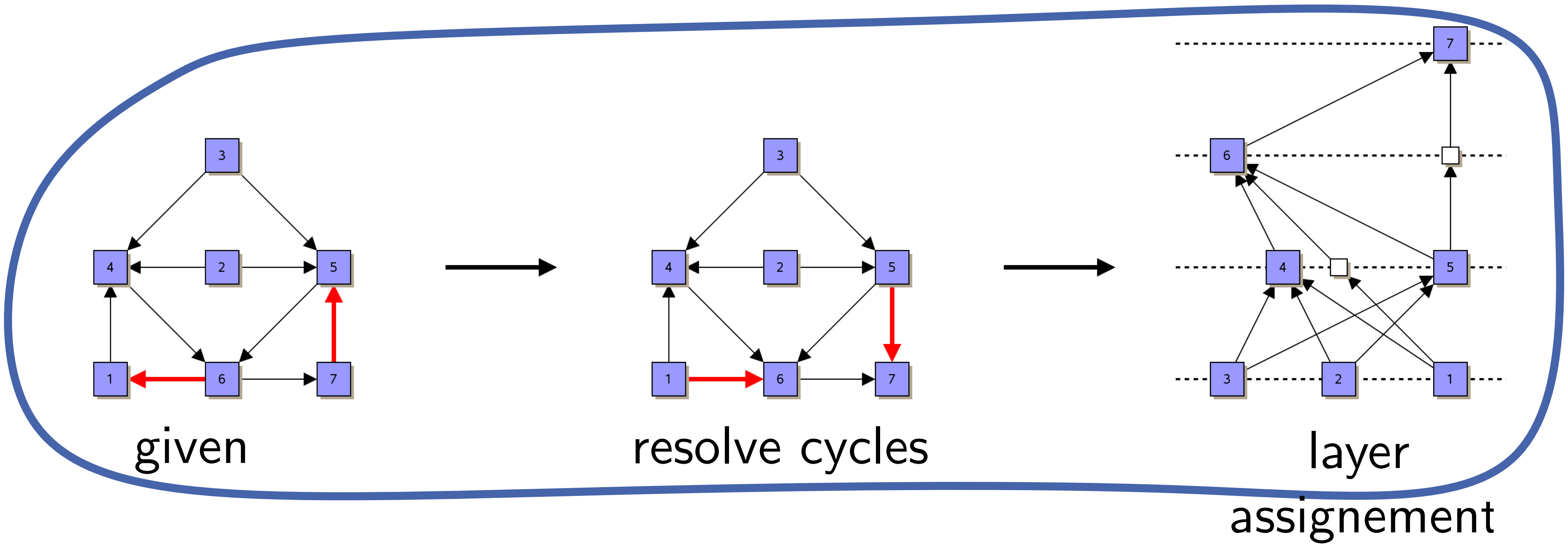
**List-Scheduling-Algorithm:**

- order jobs arbitrarily as a list $\mathcal{L}$
- when a machine is free, select an allowed job from $\mathcal{L}$; Machine is idle of there is no such job

# Summary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resolve cycles
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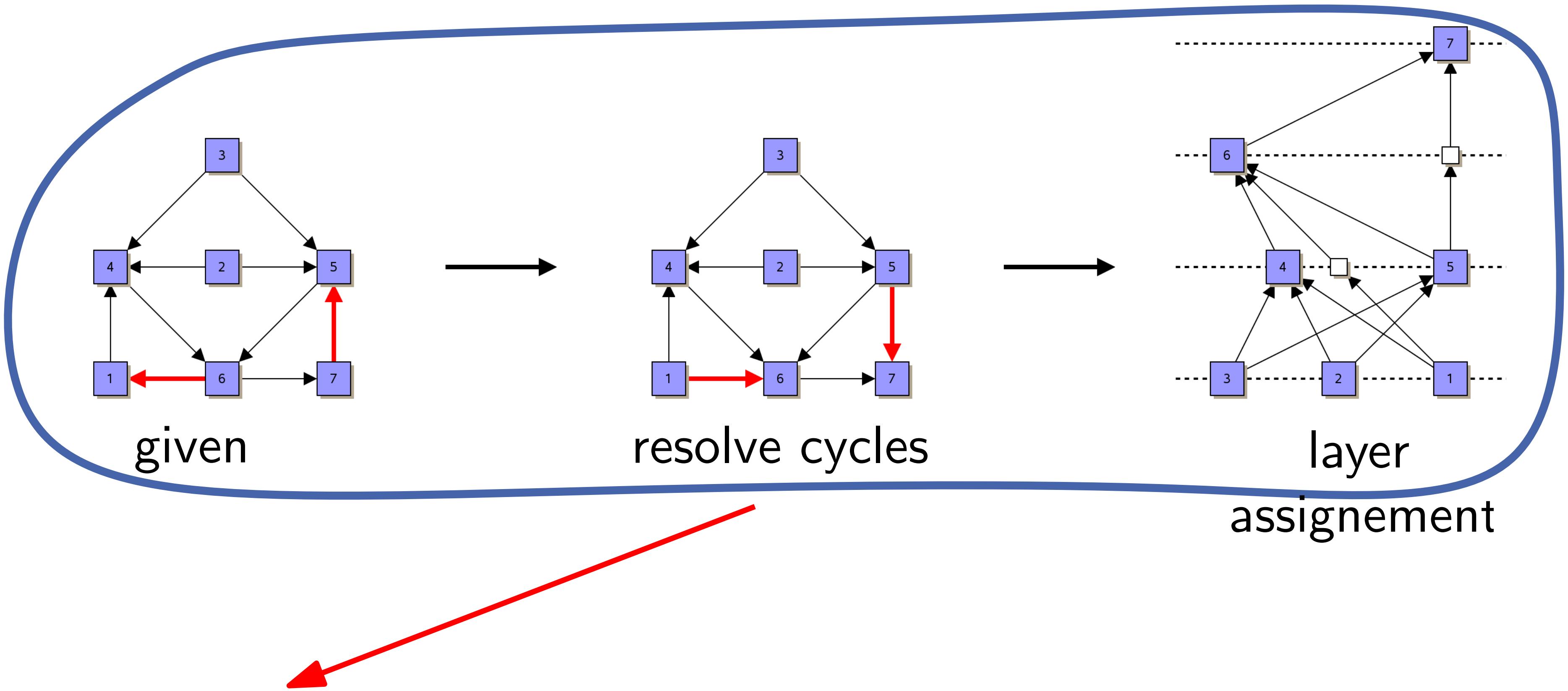
layer
assignement

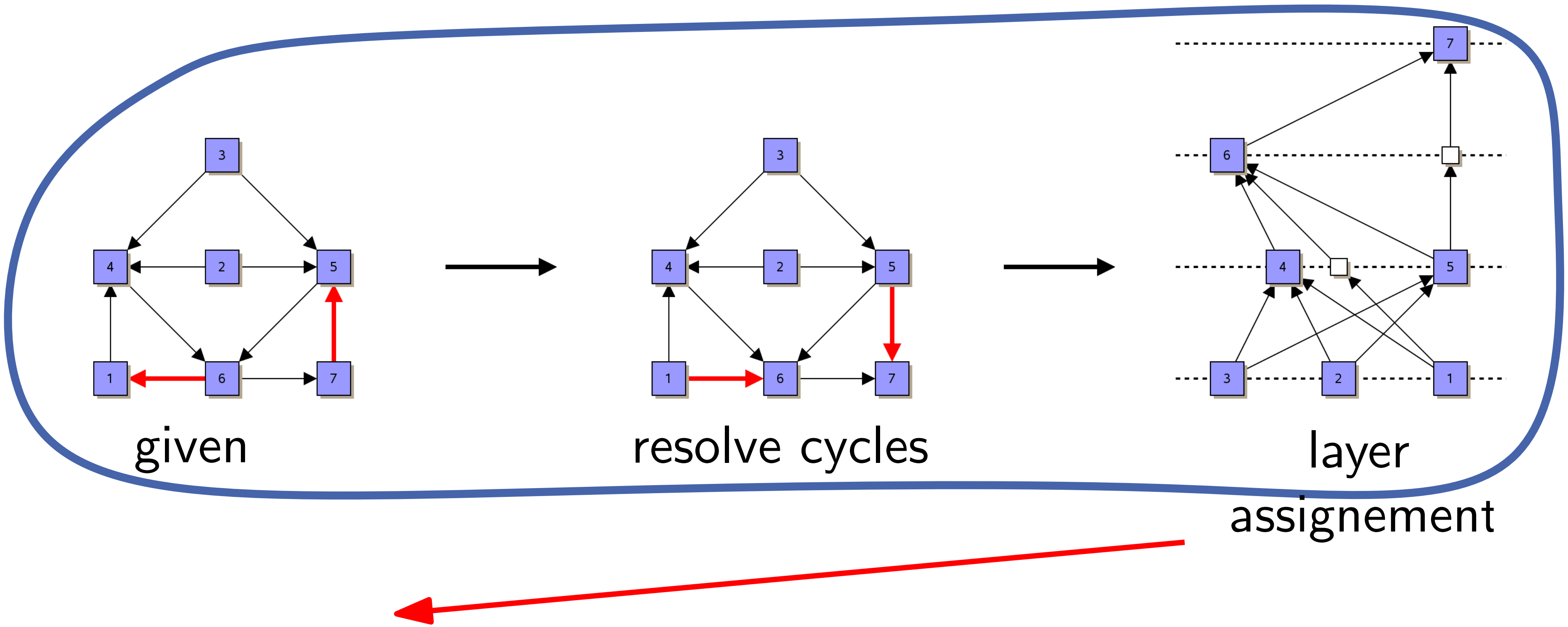
# Summary




**Resolve cycles**

- equivalent to **minimum feedback set** problem, and is NP-hard
- Heuristic with $|A'| \geq |A|/2$
- Heuristic with $|A'| \geq |A|/2 + |V|/6$

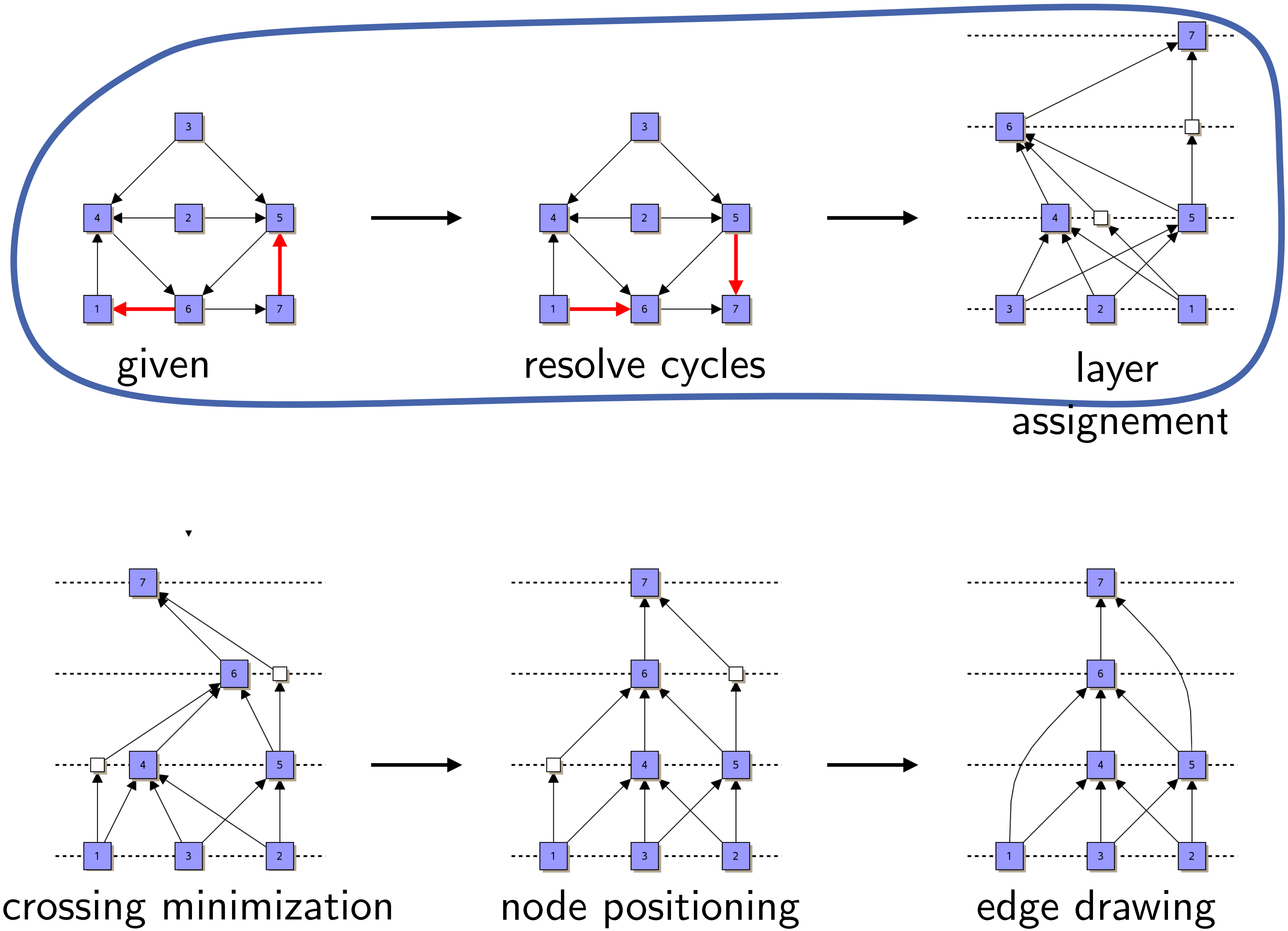
# Summary




**Layer assignement**

- Height optimization: topological numbering
- Total edge length: polynomial alg. through integer linear program
- Height optimization with fixed width: equivalent to **MPCS**. NP-hard for 3 levels. Approximation algorithm with factor $\leq 2 - \frac{1}{B}$.

# Summary


given
resolve cycles
layer
assignement
crossing minimization
node positioning
edge drawing