# Tutte's barycenter method

- Outer vertices $v_1, v_2, v_3$ are fixed at given positions.

- Each inner vertex is at the **barycenter of its neighbours**.

$$x_u = \frac{1}{\deg(u)} \sum_{v \in N(u)} x_v \quad y_u = \frac{1}{\deg(u)} \sum_{v \in N(u)} y_u \quad \text{for } u \neq v_1, v_2, v_3$$

$$\Leftrightarrow \quad \sum_{v \in N(u)} (p_u - p_v) = 0 \qquad \text{for } u \neq v_1, v_2, v_3$$

- This drawing **exists and is unique**. It minimizes the energy

$$\mathcal{P} = \sum_e \ell(e)^2 = \sum_{uv \in E} (x_u - x_v)^2 + (y_u - y_v)^2$$

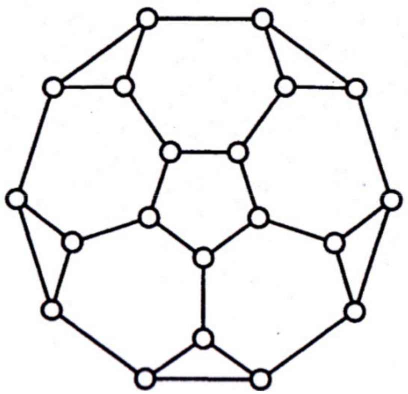
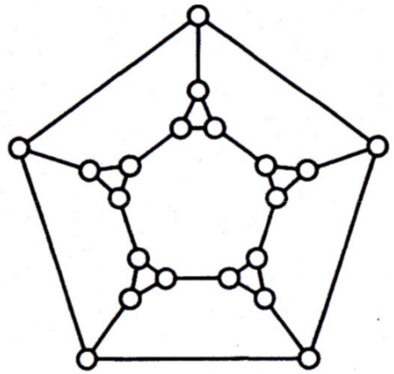
under the constraint of fixed $x_1, x_2, x_3, y_1, y_2, y_3$.

- Also a **spring embedding** where edge $e$ is a spring of energy $\ell(e)^2$.

# Advantages/disadvantages

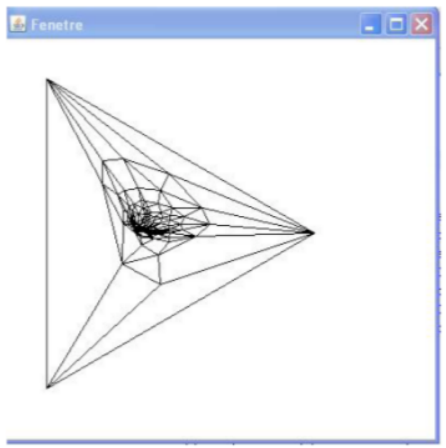
The good!

- displays the symmetries nicely
- easy to implement (solve a linear system)
- optimal for a certain energy criterion

The less good:

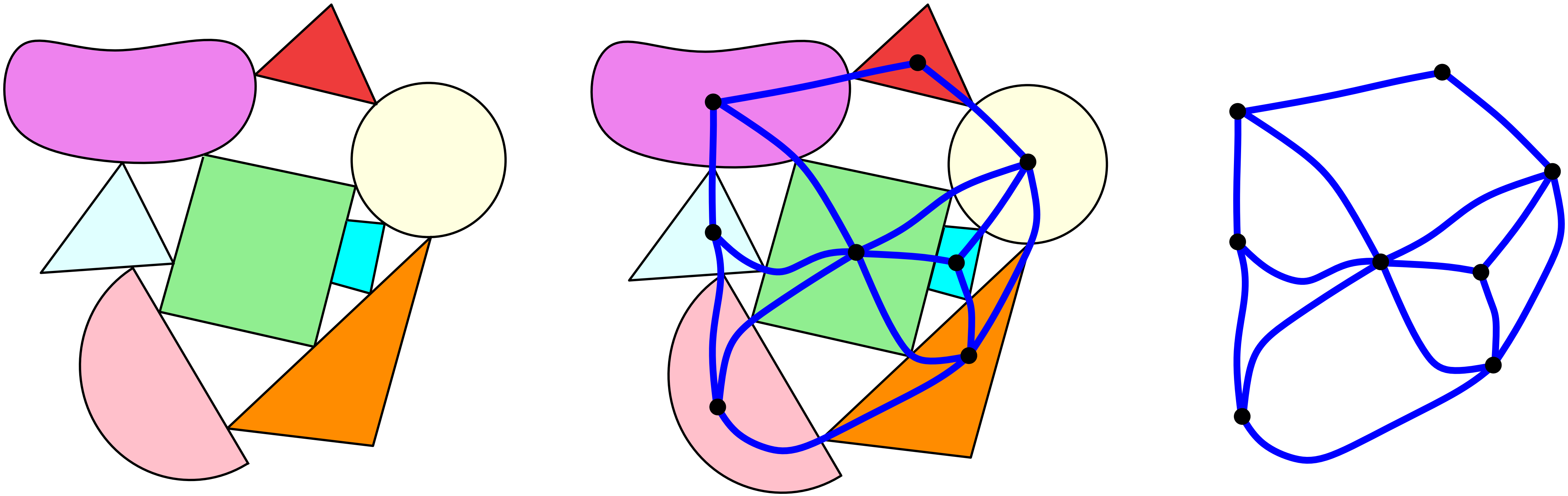
- a bit expensive computationally (solve linear system of size $|V|$)
- some very dense clusters (edges of length exponentially small in $|V|$)

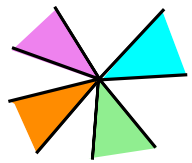


# Contact representations of planar graphs

# General formulation

**Contact configuration** = set of "shapes" that can not overlap
but can have contacts



yields a planar map (when no  )

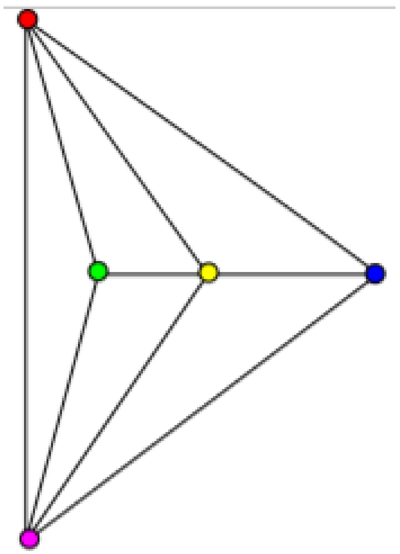
**Problem**: given a **set of allowed shapes**, which planar graphs can be realized as a contact configuration? Is such a representation unique?

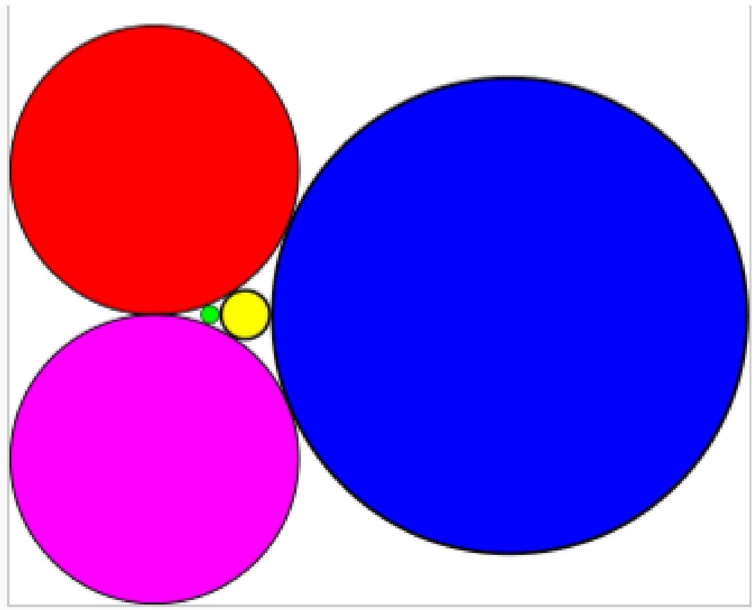
# Circle packing

**[Koebe'36, Andreev'70, Thurston'85]:**

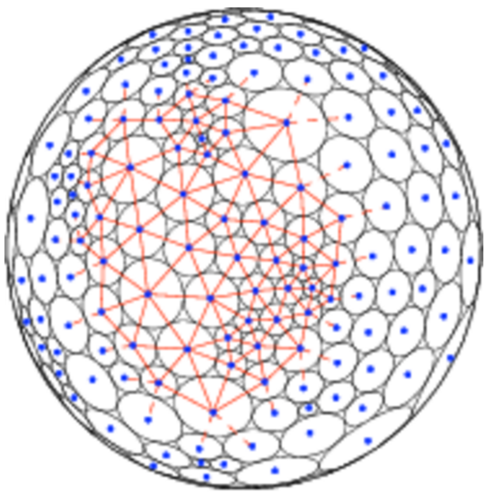
Every planar triangulation admits a contact representation by **disks**.

The representation is unique if the 3 outer disks have prescribed radius.





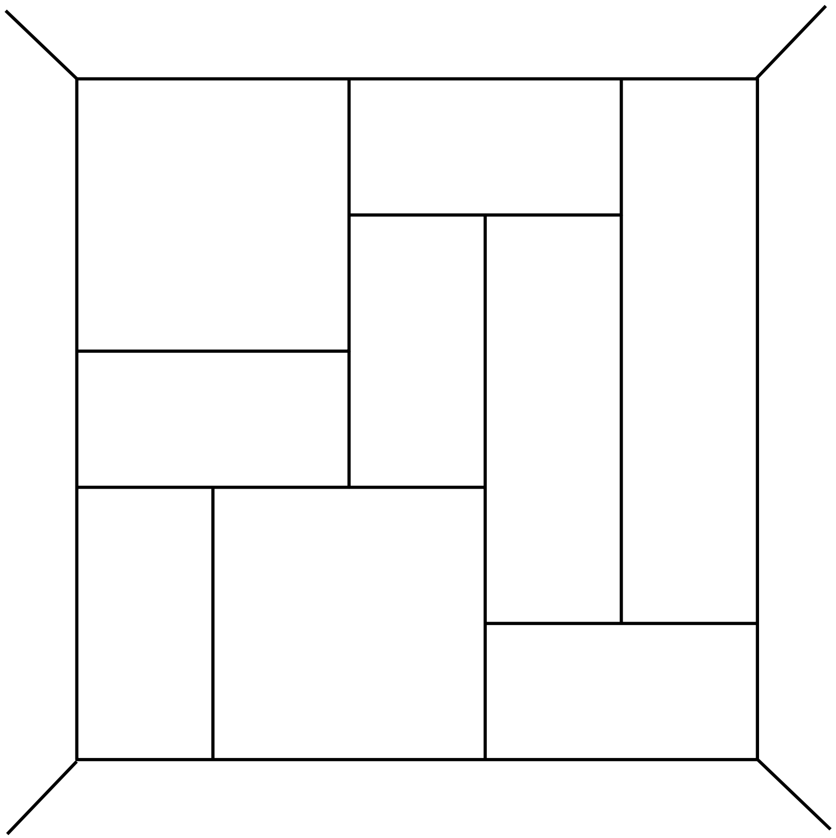
**Remark:** The stereographic projection maps circles to circles (considering lines as circle of radius $+\infty$).

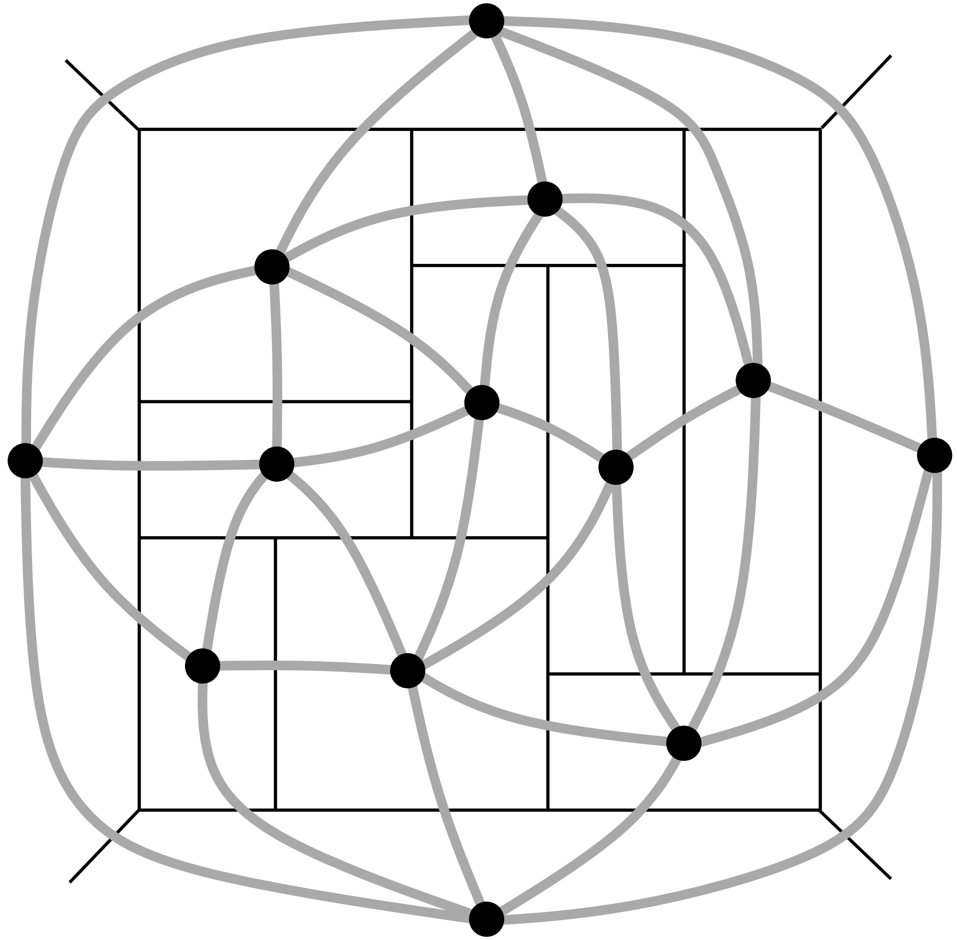


Hence one can lift to a circle packing on the sphere.

There is a unique representation where the centre of each sphere is the **barycenter** of its contact points.

# Axis-aligned rectangles in a box



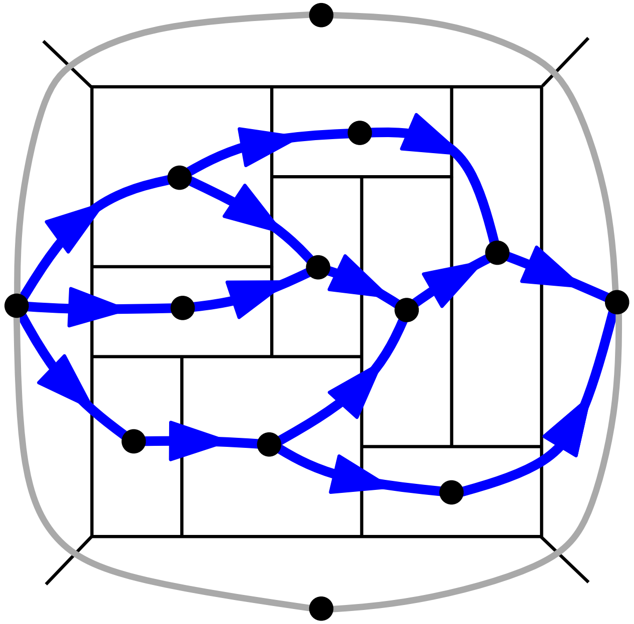


- The **rectangles** form a tiling. The contact-map is the dual map.
- This is a triangulation of the 4-gon, where **every 3-cycle is facial**.

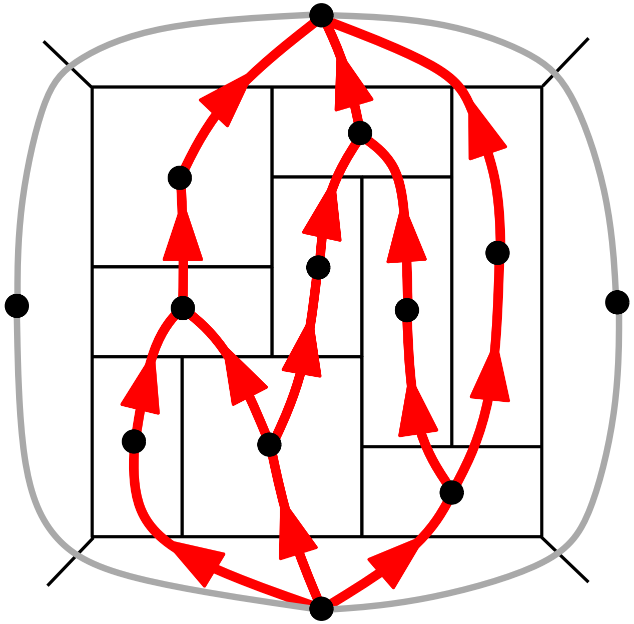
Is it possible to obtain a representation for any such triangulation?

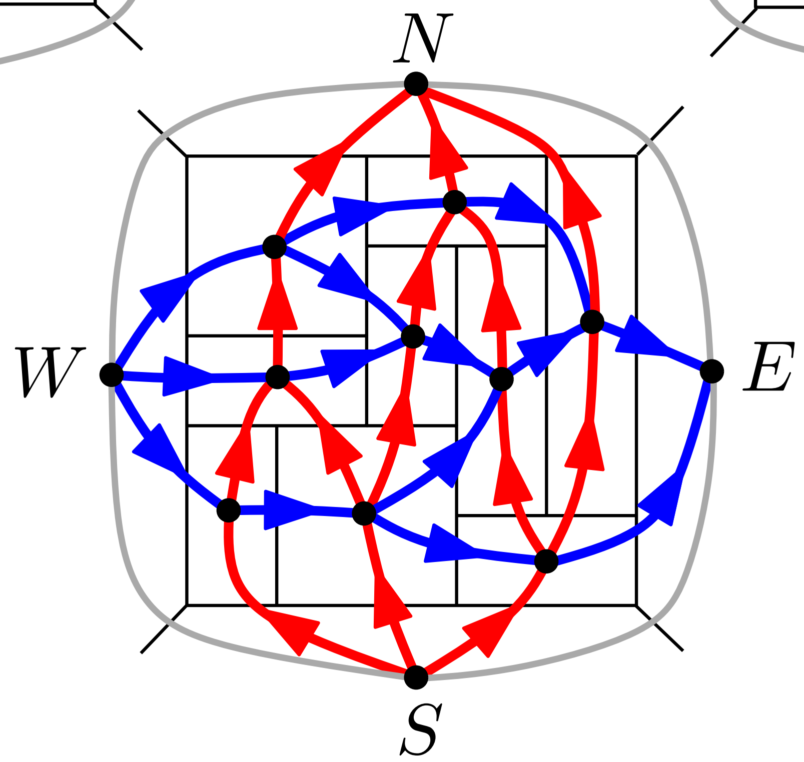
# Déjà-vu

dual for vertical edges



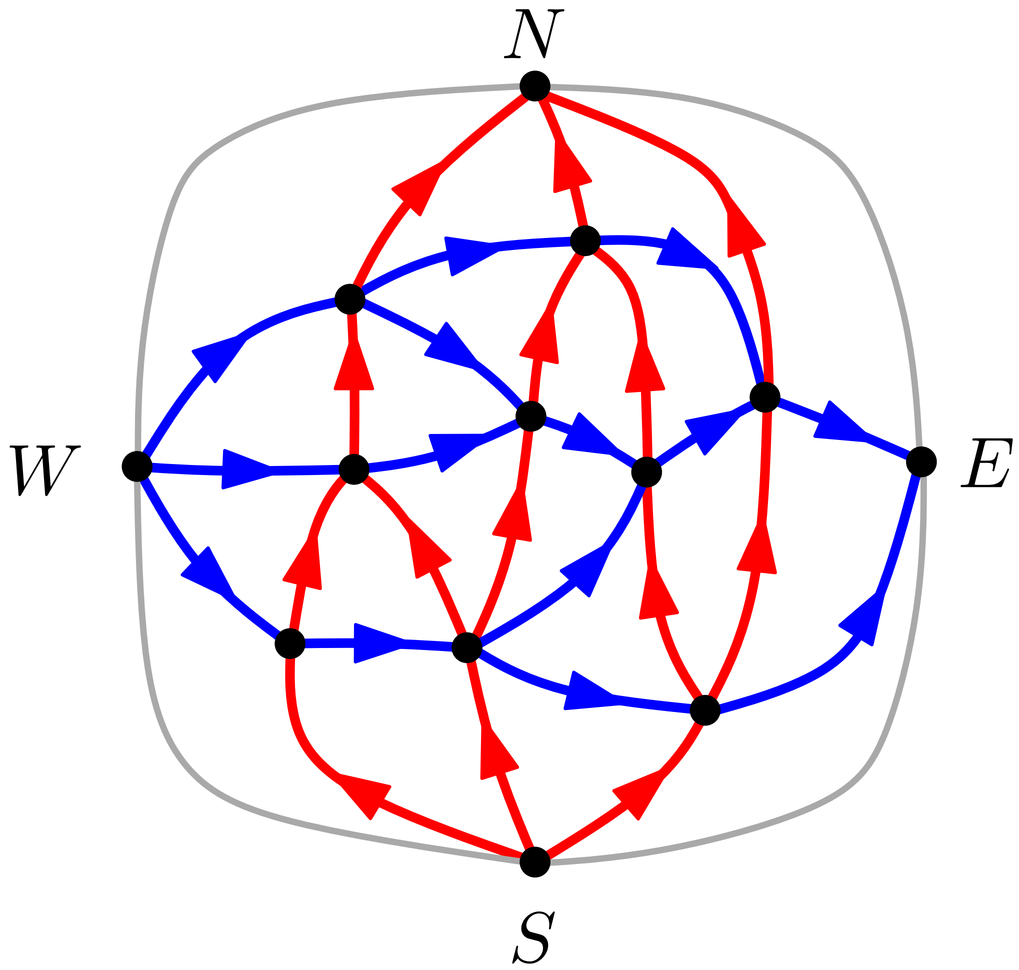
dual for horizontal edges



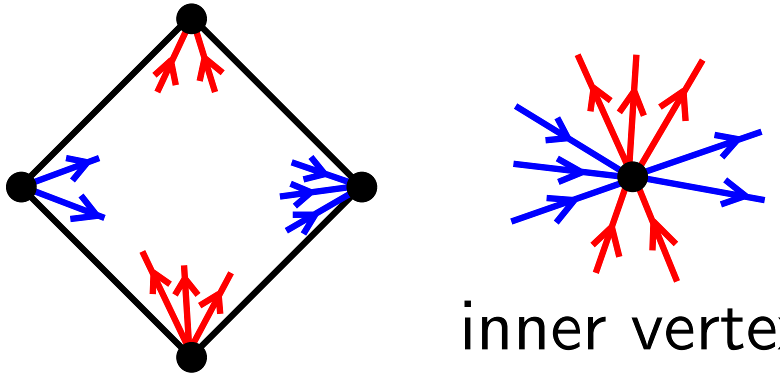


# Transversal structures

For $T$ a triangulation of the 4-gon, a **transversal structure** is a partition of the inner edges into **two transversal $s$-$t$ digraphs**.




characterized by **local conditions**:




$T$ admits a transversal structure if and only if **every 3-cycle is facial**.